Intensional Logic and the Irreducible Contrast between de dicto and de re

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Abstract
The paper deals with hot problems of current semantics that are interconnected with a fundamental question *What is the meaning of a natural language expression?* Our explication of the meaning is based on the key notion of Tichy's Transparent Intensional Logic (TIL), namely that of the logical *construction*, an entity structured from the 'algorithmic point of view' (procedure), the structure of which renders the logical mode of presentation of the ("flat") *denotatum* of an expression. Hence meaning is conceived as a concept represented by the expression, i.e. a construction in the canonical normal form. An adjustment of Materna's theory of concepts is first proposed, so that our analysis of a non-homonymous expression might be unambiguous (on the assumption of a fixed conceptual system — a common basis of our understanding each other). Synonymy, homonymy and equivalence of expressions are defined and a special case of the so-called hidden homonymy is examined. Using TIL, explicit intensionalisation enables us to precisely define the *de dicto / de re* distinction and prove two *de re* principles. Traditional hard nuts to crack, namely *de dicto / de re* attitudes and modalities are solved and we present logical reasons for not allowing β-reduction in the *de re* cases. In other words, we prove that β-conversion is *not* an equivalent transformation when working with partial functions. Logical independence of *de dicto* and */ de re* attitudes is illustrated, but a claim is proved that on an additional assumption the *de dicto* and the corresponding *de re* attitude are equivalent. Quine's example of an ambiguity in belief attribution consisting in scope for the existential quantifier is analysed and we show that it actually does concern the *de dicto / de re* distinction, this time in the supposition of the existence predicate. Last but not least, we present a "hesitant plea for partiality", though many technical difficulties (e.g. non-valid de Morgan laws) connected with partial functions are illustrated. The task of the logician is to undertake a precise analysis in order that all and only the logical consequences of our statements can be derived, even at the cost of some "technical difficulties".

Key-words: Structured meanings, concepts, synonymy, homonymy, equivalence, *de dicto / de re* supposition, *de dicto / de re* propositional attitudes, modalities, existence and *de dicto / de re*, lambda-transformation, partial functions.

1. Introduction

Logical analysis of a natural language is an exciting discipline in which we meet many problems the solution of which is a challenge not only to logicians, but also to philosophers, linguists, computer scientists, etc. Among these problems, the analysis of propositional / notional attitudes, synonymy and homonymy, using expressions in the *de dicto / de re* supposition is, probably since Frege's times, a subject of much dispute. These problems are mutually interconnected by the fundamental problem which can be characterised as that of finding an adequate answer to the following question:

What is the meaning of a natural language expression?

There are many different conceptions and approaches to answering this question, to name at least extensional semantics [Frege 1892], intensional [Montague 1974], "sentential / pragmatic" [Quine 1960, 1992], situational [Barwise, Perry 1983], and many others. Our

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approach is not classically set-theoretical but *procedural.* The meaning (sense) of a natural language expression is a *structured* abstract entity – a *procedure*, the 'algorithmic structure' of which renders the 'logical form' of an expression (by which we do not mean, unlike [Cresswell 1985], [Larson, Ludlow 1993], a *'linguistic structure'* or a *'linguistic mode of presentation',* but a *'logical mode of presentation'*), and which produces as its output (or sometimes fails to produce) the entity denoted by the expression, its *denotatum.* Unlike the latter, meaning consists of constituents ("steps"), which are again procedures (instructions). There are also many conceptions of *structured meanings*, to name at least [Cresswell 1975, 1985] and his introduction of the notion of hyperintensionality, in a way Carnap and his intensional isomorphism [Carnap 1947][1)]. An interesting attempt can be also found in [Zalta 1988] where a theory of 'abstract objects' is developed[2)].

In our opinion an adequate and powerful explication of meaning can be given by means of Transparent Intensional Logic (TIL), [Tichý 1988], [Materna 1998]. When Pavel Tichý was formulating his approach to logical analysis, later coined Transparent Intensional Logic, Montague was working out his system coined Intensional Logic (IL). Both logicians worked independently of each other, and although their approaches share some points, there are many differences between them, some of which are rather essential. For some – mostly extra-logical – reasons whose analysis is not important here, Montague's system has now become quite naturally a part of the 'mainstream' unlike TIL. The originator of the latter was sometimes a sort of *enfant terrible* who dared – as a young Czech emigrant, not yet well-known – to criticize many stars on the logical and philosophical firmament. Nonetheless, his professional standing was indisputable. The results of his analyses are most interesting and relevant, in particular in the area of logical analysis of natural language.

The fact is, however, that any paper whose author follows the Montagovians' principles does not have to repeat the important definitions and symbolic conventions to be found in Montague's works; everybody knows them. In contrast with this situation the followers of TIL have to repeat the main definitions and explain the specific symbolic notations used in TIL; only few seem to know them. Thus a negative impression might arise when somebody begins to read TIL papers: so many strange complicated "formulas", so many symbols not usually employed! An answer might be:

First, it's not that bad. There are essentially just two basic definitions, namely that of the theory of types which is a modification of Russell's ramified type hierarchy and the definition of constructions which is a modification of those known from λ-calculi. The impression of complexity perhaps arises due to the necessary 'density' of definitions.

Second, it is rewarding: the apparatus makes it possible to obtain non-trivial results. The analysis using TIL constructions is particularly fine-grained and precise, so that many "paradoxical arguments" can be easily solved. In particular, the reader of the present paper might be surprised, since my claims demonstrated therein are a little 'unexpected': Logical independence of *de dicto / de re* attitudes, but their equivalence on an additional assumption; the proof of non-equivalence of a 'general' β-reduction; *de dicto / de re* modalities and the demonstration of logical reasons for not allowing β-reduction.

Third, TIL can be characterised as a standard logic. It employs just the standard logical operators, avoiding any non-standard ones. It is *transparent, anti-contextualistic*, the meaning

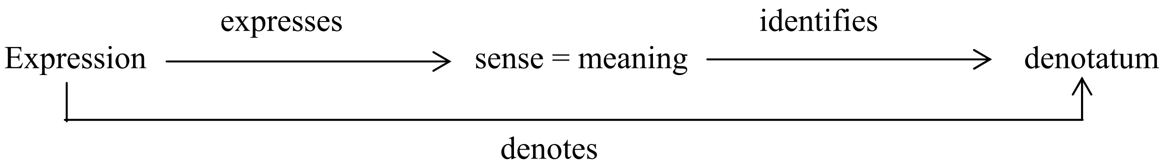
[1)] the critique of both of them can be found in [Tichý 1988] and in [Church 1954] concerning the latter

[2)] some critical comments - see [Materna 1998] of an expression is the same in all contexts. In TIL the semantic relation between an expression and what it expresses and denotes is rigid and given *a priori*, the notion of the *de dicto / de re* supposition is TIL's counterpart to reference shift and its consequent contextualism. The change of supposition is *not a shift of meaning* and the ambiguity of an expression occurring with distinct suppositions corresponds to distinct logical forms. If this were called contextualism, it would be quite an innocuous form of it. The principle of compositionality is strongly adhered to.

Yet, some features of TIL might appear as rather non-classical. We do not introduce a formal language first which would have to be interpreted afterwards. Introducing the 'language of constructions' we transparently look through it at the subject matter of our exploring which are particular *complexes - procedures.* Due to the rich 'two dimensional' ontology of entities organised in a ramified hierarchy of types, any entity of any order (including construction) can be not only used, but also safely mentioned within the theory without a danger of inconsistency. TIL is a logic of what Fritz Günthner calls 'explicit intensionalisation'. Variables ranging over possible worlds are an integral part of the TIL apparatus, which is particularly rewarding. Last but not least, in order to reflect „holes in reality" quite faithfully, to obtain a counterpart of Bolzano's *Gegenstandslosigkeit,* TIL adopts *partial functions.* Many logicians refuse the latter, for working with partial functions brings in some non-trivial difficulties. (There are also philosophical objections to partiality, e.g., in constructivism.) Still, we are convinced that introducing the category of constructions that do not construct anything ('improper constructions') is well grounded. The primary task of a logician should, after all, be an adequate analysis enabling us to deduce all and only the relevant consequences of our statements, even at the price of some 'technical difficulties'.

We do not adopt a set-theoretical semantics, for it is too coarse-grained, but adhere to a procedural fine-grained approach; using the key TIL notion of *construction,* we explicate *sense,* or *meaning* as the *concept* [Materna 1998] represented by the expression. There is much to be said for the claim that just neglecting or ignoring the notion of concept / construction as some "itinerary" leading from the expression to its denotatum brings in, at best, many pseudo-problems, and yields unsatisfactory solutions of those problems whose simple, elegant and exact solution is often quite at hand.

As a consequence of the current "state of the art", there are also terminological problems that can be characterised as mishmash. Expressions like 'sense', 'meaning', 'denotation', 'reference', etc., are often used in a vague way without an exact explication, often with different meanings. The exposition and summary of our TIL conception including an exact terminology can be found, e.g., in [Materna 1998, 1999]. A brief summary:



We identify *meaning* with Frege's sense, and meaning is explicated as a closed construction (a precise explication of an intuitive notion of *procedure*) that specifies the *concept* represented by the expression [Materna 1998], and which constructs (identifies)
the *denotatum* of the expression, which is either a first-order ("flat" set-theoretical) entity or a higher-order entity (involving constructions in its domain). The denotatum may sometimes

not exist; the identifying procedure may fail to produce anything in case of an expression representing an empty concept (expressing an improper construction), like, e.g., 'the greatest natural number'. Yet such an expression has its meaning, for we can reasonably (and truly) claim that, e.g., 'The greatest number does not exist'.

An empirical (unlike mathematical) expression always denotes an object, namely an *intension*, i.e., a function (flat mapping) from possible worlds and time points. In this case we also speak about the *reference* of the expression which is the value of the denoted intension in the actual world / time. But the expression does not "speak about" its reference; to find this reference is a matter of empirical investigation, which is out of the scope of the logical analysis of a natural language, for such an analysis is an *a priori* discipline.

The analyses and proposals of solutions of the problems mentioned above (propositional / notional attitudes, synonymy, homonymy, equivalence, *de dicto / de re*) have been outlined in [Duží 1999, 2000, 2003a, 2003b]. In the present paper we first summarize our approach (Section 2, Section 3), provide an exact definition of the *de dicto / de re* supposition (Section 4), analyze the *de dicto / de re* attitudes in Section 5 (accompanied by many examples), all of which is connected with an interesting linguistic phenomenon that can be characterized as *weak / hidden homonymy*. As a side effect of these investigations we propose an adjustment of Materna's theory of concepts [Materna 1998] (Intermezzo of Section 4). In Section 6 we analyse the predicate of existence in the connection with the *de dicto / de re* supposition, which is demonstrated by the solution to Quine's well-known Ortcutt example. Finally, Section 7 deals with the *de dicto / de re* modalities, and we demonstrate here a serious logical problem connected with partiality, namely that of a non-equivalent β-transformation and non-validity of de Morgan laws.

Before presenting our results we have first to introduce the basic notions of Transparent Intensional Logic.

2. Transparent Intensional Logic

We have stated above that the meaning of an expression can be best explicated by a closed construction that specifies a concept represented by the expression. Thus performing logical analysis of an expression consists in finding an appropriate construction. Before doing that we have first to explain what is meant by a *construction* and a *concept.* Referring for details to [Tichý 1988], [Materna 1998], we recapitulate.

A concept is an abstract structured entity, a procedure consisting of some "intellectual steps" that constructs an object out of some more primitive objects. Thus a concept consists of parts, constituents, and not only the content of a concept, i.e. its set of constituents, but primarily the *way* these constituents are bound together is significant. There may be two different concepts with exactly the same content, as was noticed already by B. Bolzano [Bolzano 1837]. *Procedure* is a timeless and spaceless entity, only its being performed is a time-consuming process and its recording is located in space. We might make a parallel with an algorithm. An algorithm is not a piece of language encoded in a (finite) alphabet, but an (effective) procedure that may be encoded in some language. It is also an "international", language-independent entity. Take, e.g., the expression '2+3', or '+(2,3)', or '[+ 2 3]'. No matter what language is used these expressions do not express the number 5 (just denote it), but a procedure which can be described as follows: Identify (take) the number 2, identify the number 3, identify the function of adding and apply this function to the given numbers (arguments). The procedure gives as its output the number 5, but, once again, this number is not the meaning (it does not consist of constituents, in this number there is no trace of 2, 3 and adding), the meaning is just the procedure itself. Consider a simple sentence

Charles calculates 2 + 3.

It denotes a relation (-in-intension) between the individual Charles and the meaning of '2 + 3'. But Charles does not calculate 5! He is trying to perform the procedure described above to arrive at the number 5, and he could have arrived at the same number by infinitely many other ways.

Similarly the expression '3 : 0' has a meaning, though it does not denote anything. Its meaning is a "road to nowhere", a procedure consisting in applying the division function to the numbers 3 and 0, which does not yield anything.

TIL constructions are such procedures. When speaking about, specifying these procedures we have, of course, to use a language, but using this "language of constructions" we speak directly about the procedures. These procedures, not the language itself, are the subject matter of our exploring. We could use any other language instead. Thus constructions, non-linguistic items, are what any two synonymous pieces of language have in common.[1]

Definition 1 *(Constructions)*

i) Atomic *constructions* are *variables*. For every type (see Def. 2) there are countably infinitely many variables which are incomplete constructions constructing an object of the respective type dependently on a valuation. We say that a variable *v*-constructs, where *v* is the parameter of valuation (a total function associating each variable with one object of the respective type).

ii) If X is any object whatsoever (even a construction), then ^0X is a *construction* called *trivialization.* ^0X constructs simply X without any change.

iii) If X is a construction that *v*-constructs a function F and $X_1,..., X_n$ *v*-construct such entities $X_1,...,X_n$ that the function F is defined on the tuple $<X_1,...,X_n>$, then $[X\ X_1\ ...\ X_n]$ is a *construction* called *composition* (in λ-calculus application), and it *v*-constructs the value of F on $<X_1,...,X_n>$, otherwise it does not construct anything and is therefore *v-improper*.

iv) Let $x_1,...,x_n$ be pairwise distinct variables and X a construction. Then $[\lambda x_1,...,x_n\ X]$ is a *construction* called *closure* (or sometimes λ-abstraction) which *v*-constructs the following function F: Let *v*' be a valuation identical with *v* up to assigning objects b_i to variables x_i ($1 \le i \le n$). Then if X is *v*'-improper, the function F is undefined on $<b_1,...,b_n>$. Otherwise the value of F on $<b_1,...,b_n>$ is the object *v*'-constructed by X.

v) Nothing is a *construction* unless it so follows from i) - iv). □

Notes: Variables and trivializations establish contact to objects which are dragged into the complex constructions, the composition and closure, so that the latter get something to work with. Our definition of variables can be conceived as an *objectual* version of the Tarskian definition. Thus variables are not letters or characters, but constructions. The letters standardly used, like *x, y, z,...,* are just names of variables. Trivialization is a special construction that might seem to be dispensable. Nevertheless, it is a very important construction, for we have to distinguish a construction (albeit primitive) of an object from the object itself, and when the object is a construction, we have to distinguish *using* this construction (e.g. $\lambda x\ [^0{>}\ x\ ^0 0]$ constructs the set of positive numbers) and *mentioning* the

[1] This feature makes TIL a platonic semantics.

construction ($^0[\lambda x\ [^0> x\ ^00]]$ constructs just $\lambda x\ [^0> x\ ^00]$). Every object can be constructed, at least by a variable or a trivialization. Trivialization is a primitive, one-step mode of presentation of some particular object. □

TIL is a logic based on a theory of types, which enables us to avoid the danger of vicious circle, and due to an infinite hierarchy of types we are not bound to a certain order (variables can range over functions, constructions, ...). Simple theory of types is, however, not powerful enough, for we need to handle constructions (concepts) as "fully-fledged" objects; we speak not only about "normal" first-order objects but also about constructions (concepts), and concepts can be not only used but also mentioned [Duží, Materna 1994]. But a variable cannot belong to its own range, a value of a function cannot be the function itself, a construction cannot construct the same construction. Hence every construction is of (belongs to) a certain type and constructs an entity of a lower type. The following definition is a generalization of Russell's ramified theory of types.

Definition 2 (*Ramified theory of types*)
Let B be a base, i.e. a collection of pair-wise disjoint non-empty sets.
T_1 (types of order 1)
i) Every member of the base B is an (elementary) *type of order 1 over B.*
ii) Let α, β_1, ..., β_n be types of order 1 over B. Then the set $(\alpha\ \beta_1 \dots \beta_n)$ of all (partial) functions (mappings) from $\beta_1 \times \dots \times \beta_n$ to α is a (functional) *type of order 1 over B.*
iii) Nothing is a *type of order 1 over B* unless it so follows from i), ii).
C_n (constructions of order n)
i) Let x be a variable ranging over a type of order n. Then x is a *construction of order n.*
ii) Let X be a member of a type of order n. Then ^{0}X is a *construction of order n.*
iii) Let $X, X_1,\dots,X_m$ be constructions of order n. Then $[X\ X_1\dots X_m]$ is a *construction of order n*
iv) Let $x_1,\dots x_m$, X be constructions of order n. Then $[\lambda x_1\dots x_m\ X]$ is a *construction of order n*.
T_{n+1} (types of order n + 1)
Let $*_n$ be the set of all construction of order n.
i) $*_n$ and every type of order n are *types of order n + 1*.
ii) If α, $\beta_1,\dots,\beta_m$ are types of order $n + 1$, then $(\alpha\ \beta_1 \dots \beta_m)$ (see T_1 ii)) is a *type of order n + 1*.
iii) Nothing is a *type of order n + 1 over B* unless it so follows from i), ii).
An object O of a type α will be called an α-object, often written as O / α.
A construction C constructing an α-object will be often written as C ... α. □

Notes:
1) The notes that follow are extremely important. The main reason of the fact that TIL did not as yet become a part of the 'main stream' in philosophical logic is that the notion of *construction* has not been understood by most members of the logicians' community. Everybody is used to work with the following scheme: There are expressions of some artificial (formal) language and they are interpreted in an inductively defined way. *Constructions in TIL are no expressions*; the way they are depicted is analogous to the way in which for example numbers are denoted: numbers are, of course, distinct from numerals, and similarly constructions are distinct from the expressions that depict them. The idea of constructions can be understood if we formulate the following question: can the semantics of complex expressions be reduced to semantics of the particular (atomic) subexpressions? In particular: Consider the simple arithmetical expression '2 + 3'. The semantics of '2', '3', '+'

is clear. Yet if we say that the semantics of the whole expression is the number 5 then something is missing - the way, that is, in which the numbers 2, 3 and the function + are composed to identify the number 5! This way of composing particular meanings to get the meaning of the whole expression is just what is defined as *construction* in TIL. Thus we can characterize constructions as *abstract procedures*: writing, e.g., $\lambda x\ [{}^0{>}\ x\ {}^00]$ (x ranging over τ) we fix a procedure that consists in abstracting over τ the application of the ($o\tau\tau$)-function > to the pair <a number, zero>; what is important is that the artificial expression can be construed as denoting the analysis of the expression *the numbers larger than zero* and that it fixes the procedure, not the result of this procedure. Thus the semantics of the above expression consists of two parts: the first corresponds to Frege's *Sinn* and is *procedural*, so it is just this *structured* construction, the second corresponds to Frege's *Bedeutung* (denotation) and is the result of that procedure, i.e., the set of positive numbers. Hence the constructions represent the objective procedures that compose the particular meanings of subexpressions to get the meaning of the whole expression.

Constructions are - unlike their outcomes - structured, with the only exception: variables. But just as the other constructions are objective procedures, variables are the same. The way they construct objects of the given type is exactly described in [Tichý 1988]: valuations offer particular infinite sequences of the objects (of the given type) and the k-th variable v-constructs the k-th member of the sequence where v is the parameter of valuations. To imagine abstract procedures is just as easy as to imagine abstract numbers as distinct from numerals. The artificial expressions that fix particular kinds of construction according to Definition 1 are not formal expressions that would wait for their interpretation: they only denote the procedures themselves (not their outcomes!). Thus the following claims are meaningful:

„ $\lambda x\ [{}^0{>}\ x\ {}^00]$ constructs the set of positive numbers"

„'$\lambda x\ [{}^0{>}\ x\ {}^00]$' contains brackets",

whereas the following claims are meaningless:

„'$\lambda x\ [{}^0{>}\ x\ {}^00]$' constructs the set of positive numbers"

„$\lambda x\ [{}^0{>}\ x\ {}^00]$ contains brackets".

Those who are still not convinced and do not conceive the notion of TIL-construction may liken the "language of constructions" to the typed λ-calculus whose terms are interpreted in a fixed "natural" way.

2) According to Def. 1, the only construction that may be improper, i.e., may fail to construct anything, is a composition. A closure always constructs a function, even if it is a "degenerated" function that does not return any value on any of its arguments (undefined on all its arguments), like, e.g., $[\lambda x\ [{}^0{:}\ x\ {}^00]]$. But Def. 1 point iii) does not distinguish two possible cases of "improperness":

a) The component X constructs a function F / ($\alpha\ \beta_1 \ldots \beta_n$) and $X_1,\ldots,X_n$ construct β_1-,...,β_n-objects, respectively, but the function F is not defined on these objects

b) Components $X, X_1,\ldots,X_n$ do not construct objects of proper types.

Def. 1 would thus not distinguish the case of a quite reasonable expression with a good sense that expresses an improper construction (but does not denote any object, like, e.g., 'the greatest natural number', '5 : 0') from the case of an expression which, though being grammatically correct, does not have any sense, i.e. does not express any construction, like 'the number 5 is a student' (category mistake) and the type system would not fulfill its type-checking role. Thus we want it to be the case that only in case a) the composition is an improper construction, whereas in case b) "composing" $[X\, X_1 \ldots X_n]$ is not a construction at all. We have to adjust Tichý's definition of composition as follows:

Definition 1', point iii)

iii) Let X be a construction that v-constructs a function F / $(\alpha\ \beta_1 \ldots \beta_n)$ and let $X_1,\ldots,X_n$ be constructions which v-construct entities $b_1 / \beta_1, \ldots, b_n / \beta_n$, respectively. Then $[X\, X_1 \ldots X_n]$ is a *construction* called *composition*. If the function F is not defined on the tuple of objects $b_1,\ldots,b_n$ or if any of $b_1,\ldots,b_n$ is not v-constructed, then the composition is v-improper (it does not construct anything). Otherwise it v-constructs the value of F on the arguments $b_1,\ldots,b_n$.

3) (Notes continued)

Quantifiers $\forall^\alpha$ (general/universal) and $\exists^\alpha$ (existential) are functional objects of type $(o(o\alpha))$, while the singularizer I^α is an object of type $(\alpha(o\alpha))$. Composition $[^0\forall^\alpha\ \lambda x\ B]$ (x...α, B...o) constructs true if λx B constructs the whole type α, otherwise false. Similarly $[^0\exists^\alpha\ \lambda x$ B] constructs true if λx B constructs a non-empty set, otherwise false. Instead of $[^0\forall^\alpha\ \lambda x\ldots]$, $[^0\exists^\alpha\ \lambda x\ldots]$ we will sometimes use the usual notation $\forall x\ldots$, $\exists x\ldots$. Singularizer $^0I^\alpha$ returns the only member of a singleton constructed by λx B, otherwise it is v-improper. Instead of $[^0I^\alpha\ \lambda x\ldots]$ we will write $\iota x\ldots$ (read: the only x such that ...).

We will also often use the classical infix notation without trivialization when writing logical connectives and identity signs to make our constructions easier to read. □

TIL is an *intensional* logic; the base used when analysing natural language expressions is the so-called *epistemic base*: the collection {ο, ι, τ, ω}, where ο is the set of truth values {True, False}, ι the universe of discourse whose members are individuals (ι being really universal, the same set regardless of possible worlds, there are no non-existing individuals), τ the set of time points (or real numbers playing the role of their surrogates) and ω the set of possible worlds (*possible world* is explicated as a maximum set of consistent, possible pre-theoretical facts). Empirical expressions denote intensions that are functions from possible worlds and time points, to a type α. Hence (α-) *intensions* are members of the type ((ατ)ω), which will be abbreviated as $\alpha_{\tau\omega}$. *Extensions* are (first-order) objects that are not intensions.

Variables $w, w^*, w_1, w_2, \ldots$ will be used as ranging over ω, variables $t, t^*, t_1, \ldots$ as ranging over τ. If X is a construction of an intension of a type $\alpha_{\tau\omega}$, we will write X_{wt} instead of $[[Xw]t]$.

Examples of intensions: *Propositions* are mappings of the type $o_{\tau\omega}$, *relations-in-intension* between members of types $\beta_1,\ldots,\beta_n$ are mappings of the type $(o\beta_1\ldots\beta_n)_{\tau\omega}$, *properties of individuals* are objects of the type $(o\iota)_{\tau\omega}$, *individual offices* (Church's individual concepts) are objects of the type $\iota_{\tau\omega}$, *magnitudes* are $\tau_{\tau\omega}$-objects.

Example of a TIL analysis. We will analyse the sentence

The most famous composer is bald.

a) <u>Type-theoretical analysis</u>

Meaningful components:

the most famous - MF, *composer* - C, *being bald* – B, *the most famous composer* - MFC

Types of the denoted objects: MF / $(\iota\ (o\iota))_{\tau\omega}$, C / $(o\ \iota)_{\tau\omega}$, B / $(o\ \iota)_{\tau\omega}$, MFC / $\iota_{\tau\omega}$

(B, C are obviously properties of individuals. MF is an intension that dependently on worlds-times associates a class of individuals with at most one individual - the most famous one, MFC is an individual office - the role an individual can play.)

b) Synthesis

The sentence claims that the holder (if any) of the MFC office at a given world-time belongs to the class of individuals who have (at that world-time) the property of being bald. To construct the office we have to combine ^{0}MF, ^{0}C - $\lambda w \lambda t\ [^0MF_{wt}\ ^0C_{wt}]$.

Applying this office to *w, t,* we obtain its holder in a given world/time - $[\lambda w \lambda t\ [^0MF_{wt}\ ^0C_{wt}]]_{wt}$.

The resulting construction is

$$\lambda w \lambda t\ [^0B_{wt}\ [\lambda w \lambda t\ [^0MF_{wt}\ ^0C_{wt}]]_{wt}\ ].$$

Or equivalently after β-reduction

$$\lambda w \lambda t\ [^0B_{wt}\ [^0MF_{wt}\ ^0C_{wt}]].$$

Note that the proposition denoted by our sentence (constructed by this construction) may have no truth-value at a given world/time W/T, for the office of the most famous composer may be vacant at W/T (if there are two or more equally famous composers, or if there are no composers). If our construction constructed a total proposition, true or false also at that W/T, it would imply [Strawson 1950] the existence of the most famous composer, which would violate the principle of a correct analysis enabling us to deduce all and only the adequate consequences of our statements. Hence the composition $[^0MF_{wt}\ ^0C_{wt}]$ may be *v*-improper, so that the office constructed by $[\lambda w \lambda t\ [^0MF_{wt}\ ^0C_{wt}]]$ is a *partial function*, and in compliance with the principle of compositionality, the composition $[\lambda w \lambda t\ [^0MF_{wt}\ ^0C_{wt}]]_{wt}$, as well as the whole $[^0B_{wt}\ [\lambda w \lambda t\ [^0MF_{wt}\ ^0C_{wt}]]_{wt}\ ]$, will in that case be *v*-improper.

c) Type-theoretical checking

$\lambda w \lambda t \quad [^0B_{wt} \quad [\lambda w \lambda t \quad [^0MF_{wt} \quad ^0C_{wt}]]\ _{wt}\ ]$

$(\iota\ (o\iota)) \quad (o\iota)$

ι

$\iota_{\tau\omega}$

$(o\iota) \quad \iota$

o

$o_{\tau\omega}$

To accomplish our brief exposition of TIL, we have to explicate the notion of *concept* [Materna 1998]. It might be obvious now that a closed construction meets the criteria that were set up in Section 1 for a logical object to be a concept. (It is a structured abstract procedure, objectual, language independent, an ”itinerary” leading from an expression to the object (if any) denoted by the expression. Analyzing an expression thus consists in specifying an appropriate construction.) Why, then, do we not identify the notion of concept with that of a closed construction? The problem consists in the fact that closed constructions are, in a way, too ”fine-grained” procedures; some closed constructions differ so slightly that they are *almost* identical. In a natural language we cannot even render their distinctness, which is actually caused by the role of λ-bound variables that do not have a counterpart in a natural language. Materna, in his monograph on concepts [Materna 1998], solves this problem by introducing an equivalence relation on the set of closed constructions, the relation of *quasi-identity* (QUID). Recapitulating briefly: This relation is induced by α- and η- transformations. Two closed constructions are α-equivalent iff they differ only by using different λ-bound variables. For instance ($\approx_\alpha / (o\ *_n *_n)$)

$^0[\lambda x\ [^0{>}\ x\ ^00]] \approx_\alpha {}^0[\lambda y\ [^0{>}\ y\ ^00]]$ (*x,y* ranging over τ).

These constructions both construct (*in the same way*) the class of numbers greater than zero, regardless of the fact which λ-bound variable is used. Two closed constructions are η-equivalent if one arises from the other by η-reduction (expansion). For instance ($\approx_\eta$ / (o $*_n *_n$)),
$^0[\lambda\, xy\, [^0{+}\, x\, y]] \approx_\eta {}^0{}^0{+}$ (+/(τττ), *x,y* ranging over τ).
Finally, closed constructions C, C' are *quasi-identical* (QUID-related) if they are either identical or there are closed constructions $C_1,\ldots,C_n$, $C=C_1$, $C'=C_n$, $n > 1$, such that every C_i, C_{i+1} are either α- or η- equivalent.
Now *concept* is defined as a set of quasi-identical closed constructions. A *concept generated by* a closed construction C is the set constructed by $\lambda c\ [^0\text{QUID}\ c\ ^0\text{C}]$, where *c* ranges over closed constructions.

A question arises: We have characterized meaning as a structured, non-set-theoretical entity, but defined a concept as a set (of constructions). Materna tries to overcome this problem by distinguishing between *using* and *mentioning* concepts [cf. Duží, Materna 1994] and by claiming that when using an expression, we use the concept represented by the expression, which means that we use any member (i.e. a closed construction) of the concept. When mentioning a concept, we construct the whole set of quasi-identical constructions, but doing so we use any representative of the concept of the mentioned concept.

Anyway, this conception is rather round about and a remedy has been proposed in [Horák 2001], consisting in identifying a *concept* with a *closed construction* in a (precisely defined) canonical *normal form*, the other (non-normal form like) QUID-related constructions pointing to the same concept. Regardless of this adjustment, the definition of the QUID relation is a crucial one, because we aim at an unambiguous analysis, unless the expression is properly homonymous. In other words, the cases of weak (hidden) homonymy should be as rare as possible, which means that if two or more closed constructions are expressed by a non-homonymous expression, these constructions *should be QUID-related.* Therefore an adjustment of the QUID relation is proposed in this paper enriching QUID by another equivalent transformation – an "innocent" β_i-equivalence, which is one of the "side-effect" results of this paper.

By way of summary, we can now answer the fundamental question put at the beginning of this paper: *The meaning of a reasonable expression is a concept represented by the expression. A concept is explicated as a (closed) construction (in the canonical, normal form) that is expressed by the expression.*

3. Synonymy, homonymy, propositional / notional attitudes

Having explicated the meaning of an expression we can now precisely define some equivalence categories on expressions:

Definition 3 *(Synonymous, equivalent, coreferential expressions)*
Expressions are *synonymous* if they have the same meaning, i.e. if they express one and the same construction or quasi-identical constructions.
Expressions are (L-)*equivalent,* if they express equivalent (congruent) constructions which construct one and the same object.
Empirical expressions are *coreferential* (coincidental) if they denote intensions that have one and the same value (reference of the expressions) in the actual world at the present time. □

In other words, synonymous expressions represent one and the same concept, whereas equivalent expressions just denote ("speak about") one and the same object (possibly in different ways).

Examples:

a) Synonymous: 'azure' – 'sky-blue', 'the man who is coming' – 'the coming man'.
b) Equivalent: 'equilateral triangle' – 'equiangular triangle',
   'It is not true that if A then B' – 'A and not B'.
   'Bill walks' – 'Bill walks and all whales are mammals'
c) Coreferential: 'Morning Star' – 'Evening Star'.

Note: Synonymous expressions are, of course, equivalent, and equivalent expressions, if empirical, are coreferential, but not vice versa. We will, however, use the term 'equivalent' for equivalent expressions not being synonymous, and 'coreferential' for coreferential expressions being neither synonymous nor equivalent.

Definition 4 (*Homonymous expressions*)
An expression E is *homonymous* (ambiguous) if it expresses two (or more) different closed constructions C_1, C_2 such that C_1, C_2 are not quasi-identical. In other words, E has more meanings.
Example: 'bank'.
Theoretically, though it should be a rare, rather "suspicious" case (for it might signal that something was wrong with the analysis), it might happen that the constructions C_1, C_2 are equivalent. In such a case we say that E is *weakly homonymous* (hidden homonymy). (The possibility of hidden homonymy has been notified in [Materna 1998, 122], where Materna analyses a classical linguistic example of two possible readings of the sentence *Flying planes can be dangerous.* This sentence is certainly ambiguous, as it is shown – there are two plausible analyses (constructions), but it is not clear whether this is really the case of hidden homonymy - as Materna claims, i.e. whether the two constructions are really equivalent. If it were so, there should be an equivalent transformation of one construction to the other one.)

Expressions specifying *propositional attitudes,* like 'to believe', 'to think', 'to know that …', 'to suppose', etc. denote generally a relation-in-intension R of an individual to the *meaning* of the embedded clause. Since we have explicated the meaning of an expression as the construction it expresses, R is an entity of type $(o\ \iota\ *_n)_{\tau\omega}$, where *n* is mostly equal to 1. This *constructional approach* to the analysis of propositional attitudes blocks the so-called *paradox of omniscience* (a substitution of an L-equivalent but non-synonymous sentence is blocked, for its meaning differs from the meaning of the original embedded sentence), and it is certainly correct, because all that is assumed about the believers (or, generally, about individuals to whom attitudes are ascribed) is that they have ideal knowledge of the language in which the report/attribution is stated, whereas it is not assumed that they know even the simplest logical / mathematical laws. Disquotation, on this idealisation, is a valid principle, and we accept it in order not to get bogged down in irrelevant problems pertaining to the transition from language to meaning. Since assent to and dissent from sentences works perfectly, our agents are already dealing with meanings. However, the constructional approach may in some cases be too restrictive. If somebody believes that London is larger than Oxford, he does not have to believe that Oxford is smaller than London. If somebody believes that A and B then he does not (!) have to believe that B and A. It does not follow, because these constructions are only equivalent, $[A \wedge B] = [B \wedge A]$, but not identical, $^0[A \wedge$

B] $\neq$ ^{0}[B $\wedge$ A]. Anyway, this constructional approach is perfectly plausible in case of attitudes to non-empirical (mathematical) sentences.

In case of attitudes to *empirical* clauses, another relation-in-intension R' (induced by R) can be taken into account [Duží, Materna 2000], namely the relation of an individual to a state-of-affairs, i.e., to the proposition constructed by the meaning of the embedded clause, R' / $(o\ \iota\ o_{\tau\omega})_{\tau\omega}$. In this case we consider some "implicit" believing, knowing, etc., whereas the relation R can be characterised as "explicit" knowing, believing, etc. [cf. Stalnaker 1999]. The believer who assents to the embedded clause certainly has to understand this sentence, i.e. to know its meaning, the respective construction. But knowing the construction does not necessarily imply knowing what entity (if any) is being so constructed. For instance, a mathematician may well know and understand the construction of Fermat's Last Theorem without *ipso facto* knowing what it constructs, which truth-value. Thus, though some iterative attitudes are valid in case of explicit attitudes (the believer does explicitly know what he/she explicitly believes, etc.), they do not have to be valid anymore in case of mixing explicit and implicit attitudes. An individual does not have to explicitly know that he/she implicitly believes, and so on. Sometimes we say "she knows it but does not know that she knows". Implicit attitudes are, however, closed under the relation of logical consequence (from which follows logical omniscience) and as such completely idle in the case of mathematical beliefs. Hence in case of explicit attitudes only ideal linguistic knowledge is assumed, allowing the believer to be a "logical / mathematical ignorant (idiot)", whereas in case of implicit attitudes she is a "logical / mathematical genius".

Notional attitudes to mathematical objects are relations-in-intension of an individual to the construction of the respective object. For example the sentence

Charles calculates 2 + 3

receives as its analysis (Calc / $(o\ \iota\ *_1)_{\tau\omega}$)

$\lambda w\, \lambda t\, [^0\mathrm{Calc}_{wt}\ ^0\mathrm{Ch}\ ^0[^0{+}\ ^02\ ^03]\ ]$

(Note that within the simple theory of types such a simple sentence could not be analysed, because we would not be able to assign a type to the object – construction $[^0{+}\ ^02\ ^03]$. It constructs a τ-object (the number 5) but cannot itself be of type τ, so it must be of a higher-order type, $*_1$ in this case.)

This constructional approach is nevertheless too restrictive in case of attitudes to empirical objects, where the attitude is generally analysed as a relation-in-intension to the denoted intension [Duží 1999]. For instance seeking is an object of type $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$ (or, as the case may be, of type $(o\ \iota\ (o\iota)_{\tau\omega})_{\tau\omega}$). The seeker intends to find the occupant of the respective office (or an individual with a given property). For example, the sentence *Charles is seeking Pegasus* is analyzed by (Seek / $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$, Ch(arles) / ι, Peg(asus) / $\iota_{\tau\omega}$) $\lambda w \lambda t\ [^0\mathrm{Seek}_{wt}\ ^0\mathrm{Ch}\ ^0\mathrm{Peg}]$. (Jespersen, in his [Jespersen 1999], uses the constructional approach even in case of seeking, which leads to some counter-intuitive results, for instance, if Charles is seeking Pegasus and a unicorn, it does not follow that Charles is seeking a unicorn and Pegasus.)

4. The de dicto / de re distinction

Distinguishing *de dicto / de re* supposition concerns expressions (and the respective constructions) denoting (constructing) functions. It is closely connected with the distinction between using and mentioning functions. A preliminary characterization would be the following:

De dicto:

An expression E is in the *de dicto* supposition in the sentence S iff the truth value of the proposition denoted by sentence S in a world *w* and time *t* is not determined (merely) by the value of the intension denoted by E in *that particular w, t* (but it is determined by the whole function).
In other words, the intension is only *mentioned* (*dictum*) and is not *used* for obtaining its value. (But the respective concept - the meaning of E is used to identify the function.)
De re:
On the other hand, we speak about the *de re* supposition when the reference of E / the value (*res*) of the denoted function "comes into play" as well: The truth value of the proposition denoted by S (in *w, t*) depends on the value of the denoted function (in *that particular w, t*).

The *de dicto / de re* distinction usually concerns only empirical expressions (i.e. expressions denoting intensions). Therefore in case of mathematical functions we will simply speak about *using vs. mentioning functions*. (However, in Section 6, when comparing the use of an existential quantifier ∃ and the use of the property of existence E, we will speak about the "*de dicto*" / "*de re*" occurrence of ∃ according to the above characterization.)

For instance, in the sentence
Cos is a periodical function
the function cosine is mentioned ("*de* dicto") whereas in the sentence
Cos(0) equals 1
cosine is used ("*de re*").

The above preliminary characterization could serve almost as a definition, but according to it the sentence S itself would be in the *de re* supposition in S, which is not correct. The sentence "speaks about" (denotes) the whole *dictum* – the proposition, never its reference (*res*) – the truth-value in the actual world / time.

Before presenting the precise definition of *de dicto / de re,* we are going to introduce the above-mentioned β_i-equivalence of constructions and propose an enrichment of the QUID relation.

***Intermezzo*: β_i-equivalence, quasi-identity**

When analysing natural-language expressions, we should follow the principle formulated by Frege, later called by P. Tichý *Parmenides' Principle*:

Ueberhaupt ist es unmöglich, von einem Gegenstande zu sprechen, ohne ihn irgendwie zu bezeichnen oder zu benennen. [Frege 1884, p.60], [Carnap 1947, §24, §26]
Or, paraphrasing freely,
The sentence speaks only about those objects whose names it contains.
The vague as this formulation is (what does it mean "speaks about", "names of"?) it can serve as a criterion:
The analysis of a sentence must combine only the concepts represented by sub-expressions of the sentence.
Consider the sentence
(HMA) The highest mountain is in Asia.
This sentence does not speak about Mount Everest (none of its sub-expression denotes Mount Everest). It speaks about objects denoted by 'the highest', 'mountain', 'being in Asia'. Combining constructions of these objects to a whole, we will receive the analysis of (HMA). But there is another reasonable sub-expression of (HMA), namely 'the highest mountain' and the concept represented by this expression should not be missing in the analysis of (HMA).

We should, properly speaking, use an adjusted version of Parmenides' principle:
The sentence speaks <u>*just*</u> *about those objects whose names it contains.*
'The highest mountain' denotes an office HM / $\iota_{\tau\omega}$ (*not* Mount Everest), 'being in Asia' denotes a property of individuals – A / $(o\iota)_{\tau\omega}$. The sentence claims that the occupant of the office HM has the property A. The preliminary analysis of (HMA) is $\lambda w \lambda t\ [{}^0A_{wt}\ {}^0HM_{wt}]$. Anyway, we will not use the simple concept ^{0}HM to construct the office HM. The construction of this office has to compose two simpler constructions of the objects denoted by 'the highest' – H and 'mountain' – M; H is a function picking (dependently on world / time) an individual from the set of individuals, the highest one, hence H / $(\iota\ (o\iota))_{\tau\omega}$. M is a property of individuals, an $(o\iota)_{\tau\omega}$-object. Hence instead of ^{0}HM we use the composed concept - $[\lambda w \lambda t\ [{}^0H_{wt}\ {}^0M_{wt}]]$, and obtain

(HMA') $\lambda w \lambda t\ [{}^0A_{wt}\ [\lambda w^* \lambda t^*\ [{}^0H_{w^*t^*}\ {}^0M_{w^*t^*}]]_{wt}\ ]$

(Renaming the inner variables *w, t* is not necessary here but makes it easier to read).
Now performing an "innocent" equivalent β_i-reduction, we get:

(HMA'') $\lambda w \lambda t\ [{}^0A_{wt}\ [{}^0H_{wt}\ {}^0M_{wt}]\ ]$

Which of the two constructions is a proper analysis of our sentence? The first one (HMA') is certainly correct but there is no serious reason for rejecting (HMA''). To obtain the occupant of the office HM we can naturally use the sub-construction $[{}^0H_{wt}\ {}^0M_{wt}]$ instead of the sub-construction $[\lambda w^* \lambda t^*\ [{}^0H_{w^*t^*}\ {}^0M_{w^*t^*}]]_{wt}$. An objection could be, however, raised: In (HMA'') there is no concept – closed construction corresponding to 'the highest mountain'. Moreover, according to [Materna 1998] the constructions (HMA') and (HMA'') are not quasi-identical, hence our simple sentence would be weakly homonymous. A minute's reflection reveals that almost all semantically self-contained composite empirical expressions would be weakly homonymous, which is certainly not desirable. The goal of the logical analysis of a natural language expression is to unambiguously indicate the meaning of the expression, unless it is inherently homonymous. Well, we could state a rule that only the non-reduced construction, (HMA') in our case, would serve as a proper analysis. In our opinion, a better solution consists in accepting both forms as adequate analyses, thus enriching the QUID relation by such an "innocent" β_i-reduction (expansion). The problem of the "missing concept" in the reduced form (HMA'') can be solved in Horák's way [Horák 2001] as follows: The sentence (HMA) represents the concept (HMA') which is the normal (canonical) form of the β_i-related constructions, and expresses also the construction (HMA'') that points to the same concept (after all, we can always expand (HMA'') into (HMA'), (HMA'') being a "shortcut" analysis). Therefore we define:

Definition 5 (*β_i -equivalence of constructions*)

Let C be a construction. By $C(x_1/y_1,..., x_n/y_n)$ we denote the result of collisionlessly [Tichý 1988, p.74, Def.17.2] replacing every occurrence of variable x_i in C by variable y_i (for $1 \leq i \leq n$). Then the construction $[[\lambda x_1...x_n\ C]\ y_1...y_n]$ is *β_i - equivalent* to the construction $C(x_1/y_1,..., x_n/y_n)$. (x_i, y_i, being of the same appropriate types).

Let CC be a construction and let $[[\lambda x_1...x_n\ C]\ y_1...y_n]$ be a sub-construction of CC. Let CC' be like CC except that instead of $[[\lambda x_1...x_n\ C]\ y_1...y_n]$ it contains $C(x_1/y_1,..., x_n/y_n)$ as a result of β_i-reducing $[[\lambda x_1...x_n\ C]\ y_1...y_n]$. Then CC and CC' are *$\beta_i$-equivalent.*

The definition of the QUID relation now comes as follows:

Definition 6 (*Quasi-identity*)

The closed constructions C, C' are *quasi-identical* (QUID-related) if they are either identical or there are closed constructions $C_1,\dots,C_n$, $C=C_1$, $C'=C_n$, $n > 1$, such that any two C_i, C_{i+1} are either α-, η-, or β_i- equivalent.

(End of *Intermezzo*)

Finally we are ready to define the *de dicto / de re* distinction.

Definition 7 (*De dicto / de re*)

Let P be a propositional construction, i.e. P is of the form $[\lambda w\lambda t\ X]$, where X constructs an object of the type o, and let us call the world / time couple constructed by *w, t* the *reporter's perspective.* Let C be an intensional construction occurring as a sub-construction within P.

We say that C is a *de re constituent of* P if there is an intensional descent of the intension constructed by C to the reporter's perspective, i.e., if C is composed with ('applied to') these (reporter's) *w, t,* or if there is a propositional construction P', β_i-equivalent to P, in which the intension constructed by C is intensionally descended to the reporter's perspective. Otherwise C is *de dicto constituent of* P.

Derivately, the same distinction applies to the expressions expressing the relevant constructions.

Examples:

1) A classical and simple one:

The American President is a democrat $[\lambda w\lambda t\ {}^0Dem_{wt}\ {}^0AP_{wt}]$

^{0}AP (The American President) is *de re*

The American President is eligible $[\lambda w\lambda t\ {}^0Elig_{wt}\ {}^0AP]$

^{0}AP (The American President) is *de dicto*

2) Consider the sentences:

(IS) *The Pope is in danger*

(ISN) *The Pope is not in danger*

('The Pope' denotes the office P / $\iota_{\tau\omega}$, 'being in danger' the property D / $(o\iota)_{\tau\omega}$)

(IS') $\lambda w\lambda t\ [{}^0D_{wt}\ {}^0P_{wt}]$ IS' - de dicto, ^{0}D, ^{0}P - de re

(ISN') $\lambda w\lambda t\ [{}^0\neg\ [\lambda w\lambda t\ [{}^0D_{wt}\ {}^0P_{wt}]]_{wt}]$ ISN' - de dicto, **IS' - de re**, ^{0}D, ^{0}P - de re

(ISN'') $\lambda w\lambda t\ [{}^0\neg\ [{}^0D_{wt}\ {}^0P_{wt}]]$ ISN'' - de dicto, ^{0}D, ^{0}P - de re

3) Consider the sentence:

The highest executive office of the USA is that of the president, not that of the king.

'The highest executive office' denotes an office occupiable by an office (hence is an intension of the second "degree") HEO / $(\iota_{\tau\omega})_{\tau\omega}$, P(resident) / $\iota_{\tau\omega}$, K(ing) / $\iota_{\tau\omega}$. Let us (for the sake of simplicity) construct the office HEO by a simple concept ^{0}HEO:

$\lambda w\lambda t\ [\ {}^0\wedge\ [{}^0=\ {}^0HEO_{wt}\ {}^0P]\ [{}^0\neg\ [{}^0=\ {}^0HEO_{wt}\ {}^0K]]\ ]$, or for short

$\lambda w\lambda t\ [\ [{}^0HEO_{wt} = {}^0P] \wedge [\neg\ [{}^0HEO_{wt} = {}^0K]]\ ]$ ^{0}HEO - *de re*, ^{0}P, ^{0}K - *de dicto*

Note that the *res* that "comes into play" in the *de re* case does not have to be only an atomic value (a member of an elementary type like an individual), but may also be a member of a *functional* type like a class, individual office, proposition, etc.

Our approach is strongly *anti-contextual (transparent).* The expressions 'the president of USA', 'the Pope', 'the highest executive office' denote the same sort of entity, an office in this case, in all the contexts. It is not true that they denote their reference (George W. Bush, John Paul II, ...) in the *de re* case, whereas in the *de dicto* case they denote their ”sense”. The notion of *de dicto / de re* supposition is thus TIL’s counterpart to reference shift and its consequent contextualism. The change of supposition is *not a shift of meaning* and the ambiguity of an expression occurring in distinct suppositions is rendered by distinct logical forms.

5. De dicto / de re attitudes

The problems connected with *de dicto / de re* attitudes are a familiar ground to almost everybody dealing with the semantics of natural language. Especially the *de re* attitudes have been a challenge. Thus Quine even claims [QUINE 1992]:

> Spelling dissolves the syntax and lexicon of the content clause and blends it with that of the ascriber's language. So long as we rest with the unanalyzed quotational form, on the other hand, the inverted commas mark an opaque interface between two ontologies, two worlds: that of the man in the attitude, however benighted, and that of our responsible ascriber of the attitude. (p. 69-70)
>
> I conclude that the propositional attitudes *de re* resist annexations to scientific language, as propositional attitudes *de dicto* do not. At best the ascriptions *de re* are signals pointing a direction in which to look for informative ascriptions *de dicto*. (p. 71)

We will show, however, that *de* re attitudes are precisely analysable using the explicit intensionalisation of TIL, which enables us to separate the two ”worlds” - the perspective of the believer and that of the reporter (ascriber). We are dealing with a fine difference between the meanings of sentences like

(P1) *Charles believes that the Pope is in danger*

(P2) *Charles believes of the Pope that he is in danger*

Some authors even claim that (P1) is ambiguous, that it can be also read as (P2). In our opinion it is not so. We can, for instance, reasonably say (it may be true) that

Charles believes of the Pope that he is not the Pope,

whereas the sentence

Charles believes that the Pope is not the Pope

cannot be true, unless our Charles is completely irrational. The sentences like (P1) and (P2) have different meanings, and their difference consists in using ‘the Pope’ in the *de dicto* supposition (P1) vs. the *de re* supposition (P2).

In the usual notation of doxastic logics the distinction is characterised as the contrast between

$B_{Charles}\, D[p]$ (*de dicto*)

$(\exists x)\, (x = p \wedge B_{Charles}\, D[x]$ (*de re*)

But there are worrisome questions [Hintikka, Sandu 1989] concerning this analysis. Where does the existential quantifier come from in the *de re* case? There is no trace of it in the original sentence. How can the two similar sentences be as different in their logical form as they are? Hintikka, Sandu propose in their [1996] a remedy by means of the Independence Friendly (IF) first order logic:

> Independence Friendly (IF) first-order logic deals with a frequent and important feature of natural language semantics. Without the notion of *independence*, we cannot fully understand the logic of such concepts as belief, knowledge, questions and answers, or

the *de dicto* vs. *de re* contrast.
They solve the *de dicto* case as above, and propose the *de re* solution with the independence indicator '/': $B_{charles}$ D[p / $B_{charles}$]
This is certainly a more plausible analysis, closer to the syntactic form of the original sentence, and the independence indicator indicates the essence of the matter; there are two *independent* questions: "Who is the pope" and "What does Charles think of that person". Of course, Charles has to have a relation of an "epistemic intimity" [Chisholm 1976] to a certain individual, but he does not have to connect this person with the office of the Pope (only the ascriber must do so). Still, the semantics of "/ $B_{charles}$" is not pellucid, and we will show that the informational independence can be precisely captured by means of TIL explicit intensionalisation without using any new non-standard operators.
(For the sake of simplicity, we will further consider only implicit attitudes, i.e. $(o\ \iota\ o_{\tau\omega})_{\tau\omega}$-objects, though the whole theory might be developed for the explicit ones as well, as it has been indicated in [Duží 1999].)

Type-theoretical analysis: B(elieve) / $(o\ \iota\ o_{\tau\omega})_{\tau\omega}$, Ch(arles) / ι, (being in)D(anger) / $(o\ \iota)_{\tau\omega}$, (the)P(ope) / $\iota_{\tau\omega}$)
(P1') $\lambda w\lambda t\ [^0B_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0D_{w^*t^*}\ ^0P_{w^*t^*}]]]$ -- ^0P (the Pope) *de dicto*
(w^*, t^* - Charles' perspective, not the reporter's)
(P2') $\lambda w\lambda t\ [\ [\lambda x\ [^0B_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0D_{w^*t^*}\ x]]]]\ ^0P_{wt}\ ]$ -- ^0P (the Pope) *de re*
(the truth value of P2 in *w, t* depends on the value of P in *this w, t* - reporter's perspective, not Charles')
The two perspectives are independent, because the two "worlds" are separated.
The analysis (P1') is straightforward: Charles has a relation to the whole proposition denoted by the 'that'-clause. The *de re* analysis of the form (P2') has been proposed in [Jespersen 2000]. How did we arrive at (P2')? Let us reformulate (P2) as the synonymous
(P2*) *The pope is such that Charles believes him to be in danger.*
(The Pope is believed by Charles to be in danger.)
Hence the Pope has the property of being believed by Charles to be in danger. Denoting this property by BCD / $(o\ \iota)_{\tau\omega}$, we get
$\lambda w\lambda t\ [^0BCD_{wt}\ ^0P_{wt}]$,
and it is obvious that ^0P and ^0BCD are *de re*. But this construction is not a sufficiently deep analysis of (P2), (P2*), because in our conceptual system [Materna 1998, 1999] there is certainly not the primitive concept ^0BCD. Though it reveals the logical structure of the *de re* sentence, we still have to identify BCD by a complex concept so as to comply with Parmenides' principle, in order to make *all* (and only) the logically significant elements of the sentence explicit. Hence BCD is constructed by:
$\lambda w\lambda t\ [\lambda x\ [^0B_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0D_{w^*t^*}\ x]]]\ ]$ (x ... ι)
Using this construction instead of ^0BCD we get:
(P2'') $\lambda w\lambda t\ [\ [\lambda w\lambda t\ [\lambda x\ [^0B_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0D_{w^*t^*}\ x]]]]]_{wt}\ ^0P_{wt}\ ]$, applying the "innocent" β_i–rule:
(P2') $\lambda w\lambda t\ [\ [\lambda x\ [^0B_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0D_{w^*t^*}\ x]]]]\ ^0P_{wt}\ ]$, applying the "general" β–rule:
(P2''') $\lambda w\lambda t\ [^0B_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0D_{w^*t^*}\ ^0P_{wt}]]]$
Now some serious questions arise: Which of these three constructions is a proper analysis of (P2), (P2*), respectively? Are (P2) and (P2*) synonymous? Are they weakly homonymous? Why did we enrich the QUID relation only with the "innocent" β_i-equivalence and *not* with

the ”general” β-transformation? Answers are not trivial and the last question will be answered in Section 5.2, where we show that β-transformation is not (!) generally an equivalent transformation when dealing with partial functions. (We will return to this problem once again in Section 7.)

Using our adjusted definition of the QUID relation we can apply the same approach to answering the above questions as proposed in Section 4: The sentence (P2*) represents the concept (P2’’) and expresses also (P2’) that points to the same concept, for (P2’’) and (P2’) are quasi-identical. We might also say that (P2’’) is in a way more accurate as an analysis, and consider (P2’) a ”shortcut” analysis. It might seem that (P2’’’) is also a proper analysis of (P2). But it is not quasi-identical with these two constructions, and it is *not* even equivalent to them, i.e. it does not construct the same proposition as (P2’). Moreover, the basic *de re* principle (existential commitment) is not respected by (P2’’’) – see the claim in Section 5.2 below. Thus we can consider (P2) and (P2*) synonymous, hence they are *not weakly homonymous*, and their proper analyses are both of the constructions (P2’’), (P2’) which are quasi-identical.

5.1. Two *de re* principles

In [Duží 2000] two important principles are formulated which hold in the *de re* cases, but not generally so in the *de dicto* cases. They are called the principles of:

a) existential presupposition (commitment)
b) intersubstitutivity of coreferential expressions

Ad a) The proposition that the ”*de re* constituent” of a sentence exists is a presupposition of the sentence. In other words, the intension denoted by the expression occurring *de re* has to be instantiated in the given world/time (the reporter’s perspective), otherwise the sentence (as well as its negation) does not have any truth value at that world/time. For instance, the sentence

Charles thinks that the King of France is in danger
(‘the King of France’ occurring *de dicto*)

is simply true or false, whereas the sentence

Charles thinks of the King of France that he is in danger
(‘the King of France’ occurring *de re*)

was true during a certain stretch of time before 1789, but does not have any truth value in the actual world now. If it were true or false then the King of France would have to exist. The respective analysis reveals this fact (the King of France – KF / $\iota_{\tau\omega}$):

$\lambda w \lambda t\, [{}^0Th_{wt}\ {}^0Ch\ [\lambda w_*\, \lambda t_*\, [{}^0D_{w*t*}\ {}^0KF_{w*t*}]]]$ (^{0}KF – *de dicto*)

(Charles has the relation of thinking to the whole proposition, regardless of whether the proposition is / is not defined in the given world/time.)

$\lambda w \lambda t\, [\, [\lambda x\, [{}^0Th_{wt}\ {}^0Ch\ [\lambda w_*\, \lambda t_*\, [{}^0D_{w*t*}\ x]]]]\ {}^0KF_{wt}\,]$ (^{0}KF – *de re*)

If the office of the King of France is not occupied in a world w and time t (that is, if the King of France does not exist), the composition ${}^0KF_{wt}$ is v-improper, which implies that the whole composition

$[\, [\lambda x\, [{}^0Th_{wt}\ {}^0Ch\ [\lambda w_*\, \lambda t_*\, [{}^0D_{w*t*}\ x]]]]\ {}^0KF_{wt}\,]$ is v- improper (Def.1’, iii)) and according to Def. 1, iv), the constructed function, proposition P, is a *properly partial* function that is undefined in those *w,t* in which the office is not occupied, for instance, in the actual world now.

But the sentence *Nobody is the King of France* is true (in the actual world now) though the

King of France does not exist and 'the King of France' occurs *de re*. Existential commitment is not met by the sentences claiming / denying existence (see Section 6).

There is an exception in the *de dicto* case where the existential principle (or even a stronger one) is valid as well, namely the case of *factiva* [Duží 1999a]. Attitudes expressed by 'knowing that …' have the presupposition of the *truth* of the embedded sentence (and consequently also existential presupposition). The sentence
Charles knows that the King of France is in danger
is neither true nor false (the same holds for the negation of the sentence) because the sentence *The King of France is in danger* is not true (it does not have any truth value, for the King of France – the *de re* component – does not exist).

Ad b) The following argument is valid:
The highest executive office of the USA is the president, not the king
The highest executive office is at the same time the most respectable office
Hence
The most respectable office of the USA is the president not the king
Type-theoretical analysis: HEO / $(\iota_{\tau\omega})_{\tau\omega}$, MRO / $(\iota_{\tau\omega})_{\tau\omega}$, P / $\iota_{\tau\omega}$, K / $\iota_{\tau\omega}$, and synthesis:

$\lambda w\lambda t\ [\ {}^0\wedge\ [{}^0{=}\ {}^0\mathrm{HEO}_{wt}\ {}^0\mathrm{P}]\ [{}^0\neg\ [{}^0{=}\ {}^0\mathrm{HEO}_{wt}\ {}^0\mathrm{K}]]\ ]$
$\lambda w\lambda t\ [{}^0{=}\ {}^0\mathrm{HEO}_{wt}\ {}^0\mathrm{MRO}_{wt}]$
$\therefore$
$\lambda w\lambda t\ [\ {}^0\wedge\ [{}^0{=}\ {}^0\mathrm{MRO}_{wt}\ {}^0\mathrm{P}]\ [{}^0\neg\ [{}^0{=}\ {}^0\mathrm{MRO}_{wt}\ {}^0\mathrm{K}]]\ ]$
Since ^{0}HEO and ^{0}MRO are *de re*, the substitution *salva veritate* is correct.

As another example of a valid argument in the *de re* case, we adduce an attitude:
Charles believes of the Pope that he is in danger
The Pope is the head of the Roman-Catholic Church.
Hence
Charles believes of the head of the Roman-Catholic Church that he is in danger.
Having defined entailment as a relation on the set of constructions, we get (P / $\iota_{\tau\omega}$, Q / $\iota_{\tau\omega}$):

$\lambda w\lambda t\ [\ [\lambda x\ [{}^0\mathrm{B}_{wt}\ {}^0\mathrm{Ch}\ [\lambda w^*\lambda t^*\ [{}^0\mathrm{D}_{w^*t^*}\ x]]]]\ {}^0\mathrm{P}_{wt}\ ]$
$\lambda w\lambda t\ [{}^0{=}\ {}^0\mathrm{P}_{wt}\ {}^0\mathrm{Q}_{wt}]$
$\therefore$
$\lambda w\lambda t\ [\ [\lambda x\ [{}^0\mathrm{B}_{wt}\ {}^0\mathrm{Ch}\ [\lambda w^*\lambda t^*\ [{}^0\mathrm{D}_{w^*t^*}\ x]]]]\ {}^0\mathrm{Q}_{wt}\ ]$

(P being the office of the Pope, Q the office of the head of the Roman-Catholic Church.)

The principle b) of the intersubstitutivity of coreferential expressions is closely connected with the question whether some *inconsistent (paradoxical) beliefs* are possible. Having the above valid premises, the reporter cannot consistently claim that Charles believes of the head of Roman-Catholic Church that he is *not* in danger. In other words, no inconsistent (contradictory) beliefs can arise in the *de re* case.

It is easy to see that in the *de dicto* case the assumptions
$\lambda w\lambda t\ [{}^0\mathrm{B}_{wt}\ {}^0\mathrm{Ch}\ [\lambda w^*\lambda t^*\ [{}^0\mathrm{D}_{w^*t^*}\ {}^0\mathrm{P}_{w^*t^*}]]]$
$\lambda w\lambda t\ [{}^0\mathrm{B}_{wt}\ {}^0\mathrm{Ch}\ [\lambda w^*\lambda t^*\ {}^0\neg\ [{}^0\mathrm{D}_{w^*t^*}\ {}^0\mathrm{Q}_{w^*t^*}]]]$
$\lambda w\lambda t\ [{}^0{=}\ {}^0\mathrm{P}_{wt}\ {}^0\mathrm{Q}_{wt}]$
do *not* entail any contradictory ('implicit') belief. (Charles can have a positive attitude to two *different* propositions, and the fact that the two offices coincide so that the propositions happen to have opposite truth-values is negligible here.)

Yet some people may have inconsistent beliefs. Frege, for instance, as a competent mathematician, certainly believed in his system without *explicitly* believing a contradiction, although, his system being inconsistent, he *implicitly* did believe a contradiction. If we define an *inconsistent belief* as a ”positive” (implicit) attitude (which he/she is usually not aware of) to the impossible proposition (the proposition that is not true in any world/time couple), we can say that inconsistent (implicit) beliefs can arise only via a set of the believer’s explicit attitudes to *constructions* the consequence of which is contradictory. (More on inconsistent beliefs - see [Jespersen 2001]).

5.2. β-reduced form of the *de re* analysis?

Consider once again the sentence
Charles thinks of the King of France that he is in danger
(‘the King of France’ occurring *de re*)
Its proper analysis is

$\lambda w\lambda t$ [[λx [${}^0Th_{wt}$ ${}^0Ch$ [λw_1 λt_1 [${}^0D_{w1t1}$ x]]]] ${}^0KF_{wt}$] (${}^0KF$ – *de re*)

This construction constructs the proposition P, a *properly partial* function that is undefined in those *w,t* in which the office KF is not occupied, because ${}^0KF_{wt}$ is *v*-improper, for instance, in the actual world now. The existential presupposition (principle a)) is respected.
Now consider the construction of the form (P2’’’), which is obtained from the above by performing β-reduction (substituting ${}^0KF_{wt}$ for *x*):

$\lambda w\lambda t$ [${}^0Th_{wt}$ ${}^0Ch$ [λw_1 λt_1 [${}^0D_{w1t1}$ ${}^0KF_{wt}$]]].

${}^0KF$ is still *de re*, which is all right, but this construction constructs another proposition, say P’, that is not a properly partial function any more; instead it is a *total* one, either true or false in any *w, t*. Hence the existential presupposition is *not* respected any more. Well, P’ ”behaves” in the same way as P in those *w, t* where ${}^0KF_{wt}$ is a proper construction (the King of France exists). Let us evaluate P’ in the actual world now. ${}^0KF_{wt}$ is now improper (the King of France does not exist), but [λw_1 λt_1 [${}^0D_{w1t1}$ ${}^0KF_{wt}$]] cannot be improper, as it *v*-constructs the ”degenerated” proposition which is undefined in all world/time pairs (for all valuations *v* that assign such possible worlds to *w* and times to *t* in which ${}^0KF_{wt}$ is improper), and the *v*-constructed proposition P’ is either *true* or *false* in the actual world now (unlike P that has *no* truth value). Partiality has disappeared, hence the latter (the β-reduced construction) is *not* a correct analysis of our sentence.

Hence we can formulate a statement claiming that the ”β-reduced form” of the analysis of a *de re* attitude is not the accurate one. Let B / $(o\ \iota\ o_{\tau\omega})_{\tau\omega}$ be the entity denoted by an attitude verb (believe, know, suppose, ...), X a construction of the ”believer” (the individual to whom the attitude is ascribed), O a construction of the office denoted by ”the F” and Prop the property ascribed to ”the F” in the *de re* supposition. Then

Claim: *The accurate analysis of a de re attitude of the form*
X believes of the F that he/she/it has the property Prop
is the non-reduced construction (DR), but the reduced (DRβ) is not correct, for it is not equivalent to the former:

(DR) $\lambda w\lambda t$ [λx [${}^0B_{wt}$ X [$\lambda w^*\lambda t^*$ [${}^0Prop_{w^*t^*}$ x]]] O_{wt}] (O - *de re*)
(DRβ) $\lambda w\lambda t$ [${}^0B_{wt}$ X [$\lambda w^*\lambda t^*$ [${}^0Prop_{w^*t^*}$ O_{wt}]]] (O - *de re*)

Proof: Let P, P’ be propositions constructed by (DR), DRβ), respectively. Then P, P’ return the same truth value in those *w,t* where O_{wt} is not *v*-improper. But P is undefined in those *w,t* where O_{wt} is *v*-improper (Def. 1, points iii), iv)), whereas P' is either true or false in those *w,t,* because $[\lambda w^*\lambda t^* [^0Prop_{w^*t^*}\ O_{wt}]]$ *v*-constructs the degenerated proposition (undefined in all *w*,t**). Since in the *de re* case there is the presupposition of the existence of 'the F', (DR) is an accurate analysis, whereas (DRβ) does not respect this presupposition. □

A seeming counter-example to the above claim is the following sentence:
The King of France believes of the Pope that he is in danger
(1β) $\lambda w\lambda t\ [^0B_{wt}\ {}^0KF_{wt}\ [\lambda w_1\ \lambda t_1\ [^0D_{w1t1}\ {}^0P_{wt}]]]$ (^0KF, ^0P – *de re*)
(1) $\lambda w\lambda t\ [\ [\lambda x\ [^0B_{wt}\ {}^0KF_{wt}\ [\lambda w_1\ \lambda t_1\ [^0D_{w1t1}\ x]]]]\ {}^0P_{wt}\ ]$ (^0KF, ^0P – *de re*)
Since $^0KF_{wt}$ is *v*-improper, for instance, in the actual world now, and $^0P_{wt}$ is not, the sentence does not have any truth value (in the actual world now), which is obviously fulfilled by (1β) and seemingly not by (1). The proposition constructed by (1) might seem to return false, because $[\lambda x\ [^0B_{wt}\ {}^0KF_{wt}\ [\lambda w_1\ \lambda t_1\ [^0D_{w1t1}\ x]]]]$ *v*-constructs the "degenerated" class, and how could it be true that the Pope belongs to such a class? But since the characteristic function of this class does not return any truth-value for any argument, the whole composition
$[\ [\lambda x\ [^0B_{wt}\ {}^0KF_{wt}\ [\lambda w_1\ \lambda t_1\ [^0D_{w1t1}\ x]]]]\ {}^0P_{wt}\ ]$ is *v*-improper (Def. 1', point iii)), which is correct.
The following table illustrates the whole state of using a **non-equivalent β-transformation**:
(O^1 a construction of the office O_1, O^2 a construction of the office O_2)
There are four possibilities of the *de re* supposition of both O^1 and O^2:
(S_1) $\lambda w\lambda t\ [\lambda x\ [^0B_{wt}\ O^1{}_{wt}\ [\lambda w^*\lambda t^*\ [^0Prop_{w^*t^*}\ x]]]\ O^2{}_{wt}\ ]$
(S_1β) $\lambda w\lambda t\ [^0B_{wt}\ O^1{}_{wt}\ [\lambda w^*\lambda t^*\ [^0Prop_{w^*t^*}\ O^2{}_{wt}]]]$
(S_2) $\lambda w\lambda t\ [\lambda x\ [^0B_{wt}\ O^2{}_{wt}\ [\lambda w^*\lambda t^*\ [^0Prop_{w^*t^*}\ x]]]\ O^1{}_{wt}\ ]$
(S_2β) $\lambda w\lambda t\ [^0B_{wt}\ O^2{}_{wt}\ [\lambda w^*\lambda t^*\ [^0Prop_{w^*t^*}\ O^1{}_{wt}]]]$

$O^1{}_{wt}$	$O^2{}_{wt}$	S_1	S_1β	S_2	S_2β
proper	proper	true/false	true/false	true/flase	true/false
proper	**improper**	**undef.**	true/false	**undef.**	undef.
improper	proper	**undef.**	undef.	**undef.**	true/false
improper	improper	undef.	undef.	undef.	undef.

5.3. "Logical contact" between *de dicto* and *de re*

The sentences (P1) involving *de dicto* attitude and (P2) involving *de re* attitude not only have different meanings, but they are not even equivalent, and, actually, there is *no logical connection* between them. *No entailment relation* between a *de dicto* attitude and the corresponding *de re* attitude generally holds. The embedded clauses denote different, independent propositions, or rather the embedded clause in the *de re* case is of a propositional form the argument of which is being filled in from the "outside", from the reporter's perspective. The fact that (P2) does not follow from (P1) is obvious; the *de re* attitude is in a way stronger, demanding the occupancy of the office (existential presupposition). Moreover, such a situation is thinkable, in which our ignorant Charles does not know who is actually the Pope and does not suppose anything about this individual, but he has read in a reliable newspaper that the Pope is in danger. Then (P1) is true while (P2) is false. It might, however, seem that (P1) followed from (P2). It is again not so. Charles may believe that Karol Wojtyla is in danger without knowing that Wojtyla is the Pope. Then the reporter may truly assert (P2) while (P1) may be false. In [Chisholm 1976] this logical contact is examined, in particular the question whether a *de re* attitude can be reduced to the corresponding *de dicto* attitude is being raised, and some criteria for such a reducibility are formulated. They could be generally summarised as follows: If there is another premise that the believer at least implicitly knows who occupies the respective office (who is the Pope, in our example), then from the *de re* attitude *and* from this additional premise the corresponding *de dicto* attitude follows. (The proof of this claim can be presented only after examining attitudes to individuals, which follows, because the attitude to an individual, the occupant of the office, plays a crucial role in this entailment.)

According to Definition 5, distinguishing between *de dicto* and *de re* occurrences is reasonable *only in case* of expressions denoting (constructions constructing) *functions*, not objects of elementary types (like individuals and numbers). Indeed, individuals or numbers are not functions, or perhaps only "quasi"-functions with zero arguments, hence *de dicto* and *de re* cases merge into one.

Not taking into account the (complicated) problem of the semantic character of proper names and considering them as being just "labels" of individuals, the following sentences are equivalent (but not synonymous):

(W1) *Charles thinks that Wojtyla is in danger*

(W2) *Charles thinks of Wojtyla that he is in danger*

(W3) *Wojtyla is a man whom Charles thinks to be in danger*

The respective analyses:

(W1') $\lambda w \lambda t\ [{}^0T_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0D_{w^*t^*}\ {}^0W]]]$ (the first sentence)

(W2,3') $\lambda w \lambda t\ [\lambda x\ [{}^0T_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0D_{w^*t^*}\ x]]\ {}^0W]$ (the second and third sentences)

Now it is unproblematic to perform β-reduction on (W2,3'), thus obtaining the *equivalent* (W1'), for there is no problem with partiality here: ${}^0W$ rigidly constructs the same individual regardless of possible worlds / times and can never be improper. The analysis of the semantic character of proper names can be found, e.g., in [Fitch 1981]. The author claims:

> "If proper names are rigid designators then they do not exhibit a meaningful *de re - de dicto* distinction in doxastic contexts."

Well, we could say that (W1') is "structurally" of the *de dicto* form, whereas (W2,3') is of the *de re* form, except that the constituent ${}^0W$ occurs (according to our definition) neither *de dicto* nor *de re*. This is not in accordance with Tichý's stipulation [Tichý 1988]. Tichý claims that proper names (or generally expressions denoting extensions) occur always *de dicto,* for they are not (and cannot be) subjected to intensional descent. Since we are digging in a way deeper and adding subtleties to Tichý's approach, we will keep our definition (and general characterization). If Charles does not (explicitly) think that John Paul II is in danger, then he simply does not know the language (linguistic incompetence). (This is certainly a drastic simplification of problems connected with the semantic character of proper names, the analysis of which is out of the scope of this paper. We just wanted to demonstrate the position of expressions denoting (not only referring to) individuals.)

(A similar approach might be applied to the analysis of attitudes with embedded sentences containing indexicals, demonstratives and other terms "denoting" individuals, like, e.g., 'that man', 'I', 'you', etc. McKinsey affirms in [McKinsey 1999 - p.521] that such sentences are structurally (logically) *de dicto,* but semantically *de re.* In our opinion the *de dicto / de re* distinction is simply not reasonable here.)

Now we are ready to prove the above claim about the reducibility of a *de re* attitude to the corresponding *de dicto* attitude with the additional premise of believer's knowing the occupant of the respective office. We will actually prove a broader statement about the mutual transferability of *de dicto* and *de* re attitudes on the assumption of the knowledge of the occupant. The proof will be again demonstrated using 'the Pope' as the paradigm, for its generalisation is obvious.

Claim: *On the assumption that the believer knows who is the occupant of the respective office, the corresponding de dicto and de re attitudes are equivalent.*

Proof: Let us assume that

(1) Charles knows that Wojtyla is the Pope
(1') $\lambda w \lambda t\ [{}^0K_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0P_{w^*t^*} = {}^0W]]]$

Since knowing is a *factivum*, the following rule is valid [Duží 1999a]:

$$\frac{[{}^0K_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0P_{w^*t^*} = {}^0W]]]}{[{}^0P_{wt} = {}^0W]} \quad (*)$$

(It holds for all *w,t* that if the upper construction *v*-constructs true so does the lower.)
Hence ${}^0P_{wt}$ is not improper, the Pope exists.
Second, we can assume that knowing implies believing (we do not "mix" explicit and implicit attitudes here):

$$\frac{[{}^0K_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0P_{w^*t^*} = {}^0W]]]}{[{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0P_{w^*t^*} = {}^0W]]]}$$

Hence we have an additional assumption
(2) Charles believes that Wojtyla is the Pope
(2') $\lambda w \lambda t\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0P_{w^*t^*} = {}^0W]]]$
We are to prove that (DR) follows from (DD) and (1), and vice versa, i.e. that (DR) and (1) imply (DD).
(DD) Charles believes that the Pope is in danger
(DD') $\lambda w \lambda t\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0D_{w^*t^*}\ {}^0P_{w^*t^*}]]]$
(DR) Charles believes of the Pope that he is in danger
(DR') $\lambda w \lambda t\ [\ \lambda x\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0D_{w^*t^*}\ x]]]\ {}^0P_{wt}\ ]$

a) (DD) ⇒ (DR)

We have (DD) and (2):
$\lambda w \lambda t\ (\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0D_{w^*t^*}\ {}^0P_{w^*t^*}]]] \wedge [{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0P_{w^*t^*} = {}^0W]]]\ )$
Since implicit attitudes are closed under the relation of logical consequence, it is true that
$\lambda w \lambda t\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [\ [{}^0D_{w^*t^*}\ {}^0P_{w^*t^*}] \wedge [{}^0P_{w^*t^*} = {}^0W]\ ]\ ]]$
$\lambda w \lambda t\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0D_{w^*t^*}\ {}^0W]\ ]]$, which is equivalent with
$\lambda w \lambda t\ [\ \lambda x\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0D_{w^*t^*}\ x]\ ]]\ {}^0W\ ]$, and since (*) is true, we have:
$\lambda w \lambda t\ [\ \lambda x\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^* \lambda t^*\ [{}^0D_{w^*t^*}\ x]\ ]]\ {}^0P_{wt}\ ]$, which we were to prove.

b) (DR) ⇒ (DD)

From (DR) and (*) it follows that Charles believes of Wojtyla that he is in danger:

$\lambda w\lambda t\ [\ \lambda x\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^*\lambda t^*\ [{}^0D_{w^*t^*}\ x]\ ]]\ {}^0W\ ]$, which is equivalent with
$\lambda w\lambda t\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^*\lambda t^*\ [{}^0D_{w^*t^*}\ {}^0W]\ ]]$. The latter and (2) give
$\lambda w\lambda t\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^*\lambda t^*\ [{}^0D_{w^*t^*}\ {}^0W]\ ] \wedge [{}^0B_{wt}\ {}^0Ch\ [\lambda w^*\lambda t^*\ [{}^0P_{w^*t^*} = {}^0W]]]$, from which we obtain (implicit believing being closed under logical consequence)
$\lambda w\lambda t\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^*\lambda t^*\ [{}^0D_{w^*t^*}\ {}^0W] \wedge [{}^0P_{w^*t^*} = {}^0W]]\ ]$, hence
$\lambda w\lambda t\ [{}^0B_{wt}\ {}^0Ch\ [\lambda w^*\lambda t^*\ [{}^0D_{w^*t^*}\ {}^0P_{w^*t^*}]\ ]$, which we were to prove.

Note that such a proof could not be performed for the case of *explicit* believing, which is not closed under logical consequence. Moreover, the assumption of *knowing* is necessary, whereas believing would not do.

5.4. Some examples

(Now we'll use only *w* without *t* for the sake of simplicity)

Charles believes that the Pope is not in danger
$\lambda w\ [{}^0B_w\ {}^0Ch\ [\lambda w_1\ [{}^0\neg\ [\lambda w_2\ [{}^0D_{w2}\ {}^0P_{w2}]]_{w1}\ ]\ ]]$ β_i - reduction:
$\lambda w\ [{}^0B_w\ {}^0Ch\ [\lambda w_1\ [{}^0\neg\ [{}^0D_{w1}\ {}^0P_{w1}]]\ ]\ ]$ ${}^0B$ - de re, $[\lambda w_1\ [{}^0\neg\ [{}^0D_{w1}\ {}^0P_{w1}]]]$, ${}^0D$, ${}^0P$ - de dicto

Charles knows that Pavel believes that Marie is a swan
$\lambda w\ [{}^0K_w\ {}^0Ch\ [\lambda w_1\ [{}^0B_{w1}\ {}^0P\ [\lambda w_2\ [{}^0S_{w2}\ {}^0M]]\ ]]]$ ${}^0K$ - de re, ${}^0B$, ${}^0S$ - de dicto

Charles thinks of the Pope that he is not the Pope
The Pope is such that Charles thinks that he is not the Pope
$\lambda w\ [\lambda x\ [{}^0T_w\ {}^0Ch\ [\lambda w_1\ [{}^0\neg\ [\lambda w_2\ [{}^0{=}\ x\ {}^0P_{w2}]]_{w1}\ ]\ ]]\ {}^0P_w]$ β_i - reduction:
$\lambda w\ [\lambda x\ [{}^0T_w\ {}^0Ch\ [\lambda w_1\ [{}^0\neg\ [{}^0{=}\ x\ {}^0P_{w1}]]]\ ]\ {}^0P_w]$ ${}^0T$ - de re

(Charles') de dicto   (ascriber's) de re

Charles believes that Pavel knows of the Pope that he is in danger
$\lambda w\ [{}^0B_w\ {}^0Ch\ [\lambda w_1\ [\lambda x\ [{}^0K_{w1}\ {}^0P\ [\lambda w_2\ [{}^0D_{w2}\ x]]]\ {}^0Pope_w]\ ]]$ ${}^0B$, ${}^0Pope$ - de re, ${}^0K$, ${}^0D$ - de dicto

The Pope is such that Charles believes that Pavel knows of him to be in danger
$\lambda w\ [\ [\lambda w\ \lambda x\ [{}^0B_w\ {}^0Ch\ [\lambda w_1\ [{}^0K_{w1}\ {}^0P\ [\lambda w_2\ [{}^0D_{w2}\ x]]]]]]_w\ {}^0Pope_w\ ]$ β_i-reduction:
$\lambda w\ [\ \lambda x\ [{}^0B_w\ {}^0Ch\ [\lambda w_1\ [{}^0K_{w1}\ {}^0P\ [\lambda w_2\ [{}^0D_{w2}\ x]]]]\ {}^0Pope_w\ ]$ ${}^0B$, ${}^0Pope$ - de re, ${}^0K$, ${}^0D$ - de dicto

Attributing self-knowledge [Castaneda 65]**:**
(He*) *The Pope believes that he (himself) is in danger.*
(He)   *The Pope believes that he is in danger.*

According to [Tichý 1971] (He*) is to be taken as making the same claim as the assertion
'I believe that I am in danger',
when made by the Pope, while (He) is to be understood as the same claim as the assertion
('he' is not to be taken as an indexical here)
'I believe that the Pope is in danger',
when made by the Pope, which does not imply 'I am in danger' as asserted by the Pope. Thus the difference between (He*) and (He) is that between the *de re* and *de dicto*, respectively:

(He*') $\lambda w\lambda t\ [\lambda x\ [{}^0B_{wt}\ {}^0P_{wt}\ [\lambda w^*\lambda t^*\ [{}^0D_{w^*t^*}\ x]]]\ {}^0P_{wt}\ ]$ ${}^0P$ (both) *de re*
(He') $\lambda w\lambda t\ [{}^0B_{wt}\ {}^0P_{wt}\ [\lambda w^*\lambda t^*\ [{}^0D_{w^*t^*}\ {}^0P_{w^*t^*}]]]]$ ${}^0P$ (the first) *de re,* ${}^0P$ (the second) *de dicto*

If we can assume that the Pope *knows who* is the Pope, (He') is reducible (see the above claim) to (He*'), and vice versa.

Another interesting example: (see [Russell 1905], [Kaplan 1968])

(Y) *Charles thinks that our yacht is longer than it is.*

Type-theoretical analysis: L(ength of) / $(\tau\iota)_{\tau\omega}$, (our)Y(acht) / $\iota_{\tau\omega}$

We have to consider a property of numbers - to be thought by Charles that the length of our yacht is greater than that number - TC / $(o\tau)_{\tau\omega}$. The sentence claims that the number - the length of our yacht has this property:

(Y') $\lambda w\, [{}^0TC_w\, [{}^0L_w\, {}^0Y_w]]$ ${}^0TC$, ${}^0L$, ${}^0Y$ - *de re*

We have to construct the property TC: $\lambda w\, [\lambda x\, [{}^0T_w\, {}^0Ch\, [\lambda w_1\, [{}^0{>}\, [{}^0L_{w1}\, {}^0Y_{w1}]\, x]]]]$ (x ... τ)

(Y'') $\lambda w\, [\, [\lambda w\, [\lambda x\, [{}^0T_w\, {}^0Ch\, [\lambda w_1\, [{}^0{>}\, [{}^0L_{w1}\, {}^0Y_{w1}]\, x]]]]]_w\, [{}^0L_w\, {}^0Y_w]\,]$ β_i - reduction
(Y''') $\lambda w\, [\, [\lambda x\, [{}^0T_w\, {}^0Ch\, [\lambda w_1\, [{}^0{>}\, [{}^0L_{w1}\, {}^0Y_{w1}]\, x]]]]\ \ [{}^0L_w\, {}^0Y_w]\,]$

${}^0L$, ${}^0Y$ (the former, Charles' perspective) - *de dicto*, ${}^0L$, ${}^0Y$ (the latter, reporter's) - *de re*

Now there is a question: Is performing the β-reduction on (Y''') "dangerous" here? In other words, can the composition $[{}^0L_w\, {}^0Y_w]$ be improper? The answer is yes, our yacht does not have to actually exist, and in such a case the sentence (Y) is not "reasonable", it is neither true nor false, which is rendered by the correct (Y'''). It is parallel to claiming that Charles thinks that the King of France is taller than he actually is. Such a sentence is nowadays not "reasonable", it does not have any truth-value (otherwise it would imply that the King of France does exist). But after performing the β-reduction we would obtain (note that renaming the inner variable *w* - Charles' perspective is necessary here, otherwise there would be a collision of variables and we'd obtain a contradictory attitude)

(Y'''') $\lambda w\, [\, [{}^0T_w\, {}^0Ch\, [\lambda w_1\, [{}^0{>}\, [{}^0L_{w1}\, {}^0Y_{w1}]\, [{}^0L_w\, {}^0Y_w]]]]\,]$

(Charles') *de dicto* (reporter's) *de re*

that again constructs a *total* proposition, which is not correct.

What follows are examples that are usually solved by means of some non-standard operators (*backwards-looking operators* or *informational independence* operator [Hintikka, Sandu 1989]). We present here a solution using in a way standard means (just the TIL explicit intensionality and the trivialisation are "non-standard" here).

1st example:

(J) John believes that there are people who hate him, but some of them actually love him.

We first construct the property BJ / $(o\iota)_{\tau\omega}$ - being an *x* that John believes that *x* hates John (H(ating) / $(o\,\iota\,\iota)_{\tau\omega}$):
$\lambda w \lambda t\, [\lambda x\, {}^0B_{wt}\, {}^0J\, [\lambda w^* \lambda t^*\, [{}^0H_{w^*t^*}\, x\ J]]]$

The analysis of (J) comes as follows:

(J') $\lambda w\lambda t$ ($[{}^0B_{wt}\ {}^0J\ [\lambda w_1\lambda t_1\ {}^0\exists x\ [{}^0H_{w1t1}\ x\ {}^0J]]] \wedge {}^0\exists x$ ($[{}^0BJ_{wt}\ x] \wedge [{}^0L_{wt}\ x\ {}^0J]$))

Note that the truth-value of the proposition does not depend on the value the first $\exists$ returns - John can believe that some people hate him even if there are no such people, but such people do exist who are believed by John ... and who love him. We might say that the first ${}^0\exists$ "occurs in *de dicto* context", whereas the second one "occurs in *de re* context" (according to our general characterisation - the beginning of Section 4).
Substituting the above construction of the property BJ for ^{0}BJ, we obtain

(J'') $\lambda w\lambda t$ ($[{}^0B_{wt}\ {}^0J\ [\lambda w_1\lambda t_1\ {}^0\exists x\ [{}^0H_{w1t1}\ x\ {}^0J]]] \wedge$
${}^0\exists x$ ($[\ [\lambda w\lambda t\ [\lambda x\ {}^0B_{wt}\ {}^0J\ [\lambda w^*\lambda t^*\ [{}^0H_{w^*t^*}\ x\ J]]]]_{wt}\ x] \wedge [{}^0L_{wt}\ x\ {}^0J]$)),

and performing β_i-reduction

(J''') $\lambda w\lambda t$ ($[{}^0B_{wt}\ {}^0J\ [\lambda w_1\lambda t_1\ {}^0\exists x\ [{}^0H_{w1t1}\ x\ {}^0J]]] \wedge$
${}^0\exists x$ ($[\ {}^0B_{wt}\ {}^0J\ [\lambda w^*\lambda t^*\ [{}^0H_{w^*t^*}\ x\ J]]] \wedge [{}^0L_{wt}\ x\ {}^0J]$)).

^{0}B (both) - *de re*, ^{0}L - *de re*, ^{0}H (both) - *de dicto*, ${}^0\exists$ (first) - "*de dicto*", ${}^0\exists$ (second) - "*de re*"

2nd example:

(M) Once Mary did believe that she would be happy now.

Now we will make use of the **temporal dimension** of TIL:

(M') $\lambda w\lambda t\ [{}^0\exists t_1\ ((t_1 < t) \wedge [{}^0B_{wt1}\ {}^0M\ [\lambda w^*\ \lambda t^*\ [{}^0Happy_{w^*t}\ {}^0M]]]\ )$

^{0}B - "*de re* modally", "*de dicto* temporally", ^{0}Happy - "*de dicto* modally", "*de re* temporally"
(Note that 'now' refers to reporter's present time-moment, which is rendered by ${}^0Happy_{w^*t}$.)
Well, we might even distinguish a modal (with respect to w) de *dicto / de re* occurrence and temporal (with respect to *t*) *de dicto / de re* occurrence. Of course, if the constituent is „only" modally or „only" temporally *de dicto*, then it is not *de re* any more and the two *de re* principles do not hold. To illustrate this situation, we will analyse Tichy's example:

3rd example:

Consider the sentence ([Tichý 1986, p.263])

(F) My next-door neighbour is frequently sick

This sentence is, in our opinion, actually ambiguous. It can be read as

(F1) It is frequently the case that my next-door neighbour is sick
or
(F2) My next-door neighbour is such that he is frequently sick.
If we have another statement that
(G) My next-door neighbour is the mayor
then from (F2) and (G) it does follow that
(H2) The mayor is such that he is frequently sick,
whereas from (F1) and (G) we cannot infer (H2) nor
(H1) It is frequently the case that the mayor is sick,
for if my neighbour's mayorship is sufficiently brief (or if the occupant of the office of my neighbour is not the same person within some time interval), (F1) and (G) may well be true but the "conclusions" (H1), (H2) false. The reason for this consists again in the *de dicto / de re* distinction, the **temporal** one this time:

(my)N(eighbour) / $\iota_{\tau\omega}$, (the)M(ayor) / $\iota_{\tau\omega}$, (being)S(ick) / $(o\iota)_{\tau\omega}$, FR(equently) / $((o\ (o\tau))\ \tau)$
(FR is the function which takes every time-moment T to the class of time intervals which are frequent in T, e.g. within six months of T at least once a week)
(F1') $\lambda w\lambda t\ [{}^0FR_t\ [\lambda w^*\lambda t^*\ [{}^0S_{w^*t^*}\ {}^0N_{w^*t^*}]]_w\ ]$ or after β_i-reduction
(F1'') $\lambda w\lambda t\ [{}^0FR_t\ [\lambda t^*\ [{}^0S_{wt^*}\ {}^0N_{wt^*}]]\ ]$ $\quad {}^0N$ - *de dicto* **temporally**

(F2') $\lambda w\lambda t\ [\lambda x\ [{}^0FR_t\ [\lambda t^*\ [{}^0S_{wt^*}\ x]]\ ]\ {}^0N_{wt}\ ]$ $\quad {}^0N$ - *de re*
(G') $\lambda w\lambda t\ [{}^0N_{wt} = {}^0M_{wt}\ ]$
$\therefore$
(H2') $\lambda w\lambda t\ [\lambda x\ [{}^0FR_t\ [\lambda t^*\ [{}^0S_{wt^*}\ x]]\ ]\ {}^0M_{wt}\ ]$
Hence the principle of intersubstitutivity of coreferential expressions holds for (F2) but it does not hold for (F1).
Similarly, from (F2) it does follow that there is an individual who is frequently sick,
$\lambda w\lambda t\ \exists z\ [\lambda x\ [{}^0FR_t\ [\lambda t^*\ [{}^0S_{wt^*}\ x]]\ ]\ z]$ $(z \ldots \iota)$
whereas the existential commitment does not hold for (F1). We cannot infer from (F1) that there is an individual that is frequently sick (e.g. if the office of my neighbour often changes its holder, and sometimes it can even be vacant).
Nevertheless, the following argument is valid:
(F1'') $\lambda w\lambda t\ [{}^0FR_t\ [\lambda t^*\ [{}^0S_{wt^*}\ {}^0N_{wt^*}]]\ ]$
$\lambda w\forall t\ [{}^0N_{wt} = {}^0M_{wt}\ ]$
$\therefore$
$\lambda w\lambda t\ [{}^0FR_t\ [\lambda t^*\ [{}^0S_{wt^*}\ {}^0M_{wt^*}]]\ ]$
(If N and M were such offices that would have the same holder in a given world eternally, e.g. the office of God and the office of the most perfect being, we might use such an argument.)

6. Existence and de dicto / de re.

We have stated above that there is an important difference between ascribing belief attitudes in the *de re* way (from reporter's - ascriber's perspective) and in the *de dicto* way (fully in the competence of the person to whom the attitude is ascribed). As a typical example of this difference, Quine's well-known example [Quine 1956] is often put forward. According to Quine the following sentence is ambiguous:

Ortcutt believes that someone is a spy

It can be read in two ways. It can simply mean
(1) Ortcutt believes that there are spies (i.e., that spies exist)
or it can convey a more interesting piece of information,
(2) Someone in particular is believed by Ortcutt to be a spy.
The difference between these two sentences is (according to Quine) characterised by the scope of the existential quantifier (small scope in (1), large scope in (2)):
(1*) Ortcutt believes: $\exists x$, x is a spy
(2*) $\exists x$, Ortcutt believes that x is a spy
But the difference does not consist only in the scope of the quantifier, for at least equally important is the *de dicto / de re* difference. Sentence (1) expresses Ortcutt's relation to a *dictum*, the whole propositional content, whereas (2) expresses Ortcutt's relation to a *res*, individual, about whom he has a *de re* belief.

In the previous section we stated that the *de dicto / de re* distinction is not reasonably applicable in case of constructions constructing (expressions denoting) individuals. About which component of the above sentences can we thus claim that it has the *de dicto* occurrence in (1) and *de re* occurrence in (2)? It cannot be the variable x that v-constructs an individual (after all, there is no name of the bound variable x in the sentences), but must be ”something” corresponding to 'someone', i.e. existence. Therefore, before presenting the TIL analysis of (1) and (2), we have to say a few words about the way TIL analyses *existence*.

As Tichý [1979] claims, existence is a perfectly good property, however, not of individuals but of offices (or generally intensions), the property of being occupied (having a non-empty ”population”). Thus existence E is an object of type $(o\ \alpha_{\tau\omega})_{\tau\omega}$, where α is usually ι or $(o\iota)$. Compare two simple sentences:

(E1) Spies exist (There are spies)

(E2) Somebody is a spy

They are certainly equivalent, but are they also synonymous? In which *de dicto / de re* supposition does the component 'spy' ('spies') occur?

The sentence (E1) claims that the property of being a spy ($S / (o\iota)_{\tau\omega}$) has the property of existence ($E / (o\ (o\iota)_{\tau\omega})_{\tau\omega}$). Hence we simply obtain

(E1') $\lambda w \lambda t\ [{}^0E_{wt}\ {}^0S]$ where the component ${}^0S$ ('spies') occurs *de dicto.*

Traditionally, when analysing existential sentences we use an existential quantifier. Therefore we can ask: ”Does the expression 'exist' ('there are') represent the simple concept ${}^0E$, or in other words, is ${}^0E$ a primitive concept of our conceptual system [Materna 1998]?” Most probably not, but we can construct the property E using the existential quantifier $\exists / (o\ (o\iota))$:

$\lambda w \lambda t\ [\lambda c\ [{}^0\exists\ \lambda x\ [c_{wt}\ x]]]$ (where $x \ldots \iota$, $c \ldots (o\iota)_{\tau\omega}$)

Using this definition of E instead of ${}^0E$, we obtain:

(E1'') $\lambda w \lambda t\ [\ [\lambda w \lambda t\ [\lambda c\ [{}^0\exists\ \lambda x\ [c_{wt}\ x]]]]_{wt}\ {}^0S\ ]$ (${}^0S$ – *de dicto*)

and after performing the innocent β_i-reduction we get a quasi-identical

(E1''') $\lambda w \lambda t\ [\ [\lambda c\ [{}^0\exists\ \lambda x\ [c_{wt}\ x]]]\ {}^0S\ ]$ (${}^0S$ – *de dicto*).

The sentence (E1) thus represents the concept (E1'') and expresses also (E1''') that points to the same concept. The component ${}^0S$ ('spies') occurs *de dicto.* The sentence (E2) claims that there is an individual (${}^0\exists\ x$) with the property of being a spy. Thus the most natural analysis of (E2) is

(E2') $\lambda w \lambda t\ [{}^0\exists\ \lambda x\ [{}^0S_{wt}\ x]]$, or for short $\lambda w \lambda t\ [{}^0\exists x\ [{}^0S_{wt}\ x]]$, where ${}^0S$ ('spy') occurs *de re*.

It is clear that (E1''') and (E2') cannot be one and the same concept (they are not quasi-identical and the component ${}^0S$ occurs *de dicto* in the former, *de re* in the latter), hence (E1), (E2) are *not synonymous*. But they are, of course, equivalent; β-reduction on (E1''') yields (E2'), which is this time an equivalent transformation because there is no problem with partiality (${}^0S$ cannot be improper). Anyway, we just stated another reason for not allowing to enrich the QUID relation by the ”general” β-reduction even in such a case when it *is* an equivalent transformation (the substituted construction cannot be improper): The supposition of a constituent can be changed by a β-reduction, and how could it be that one and the same constituent would occur both *de dicto* and *de re* within one and the same concept?

An analogous semantic distinction can be observed when analysing, e.g., the following two sentences that are again *not* synonymous:

Pegasus does not exist

$\lambda w \lambda t\ [\neg\ [{}^0E_{wt}\ {}^0Peg]]$ ${}^0Peg$ - *de dicto* $(E / (o\ \iota_{\tau\omega})_{\tau\omega}$

"Nobody" (no individual) is Pegasus

$\lambda w \lambda t\ [\neg\ {}^0\exists \lambda x\ [x = {}^0\mathrm{Peg}_{wt}]]$ ^{0}Peg - *de re*

Note that though ^{0}Peg ('Pegasus') is *de re* in the latter, the sentence is true in those world/times where Pegasus does not exist (${}^0\mathrm{Peg}_{wt}$ *v*-improper, $\lambda x\ [x = {}^0\mathrm{Peg}_{wt}]$ *v*-constructs again the 'degenerated' class). ***The existential presupposition is not valid, of course, in case of sentences claiming / denying existence.*** (See also [Duží, Materna 1994], Materna 1998, pp.104-5].)

Let us return to the Ortcutt example. Quine's sentence could be understood (taking into account very tolerant readings) in four variants:

(O1) Ortcutt believes that someone is a spy
(O2) Ortcutt believes that spies exist
(O3) Someone is believed by Ortcutt to be a spy
(O4) There are people who are believed by Ortcutt to be spies

(O1) and (O2) are obviously *de dicto* attitudes, whereas (O3) and (O4) are *de re*. Since attitudes *de re* are notoriously more intractable than attitudes *de dicto*, let us begin with (O1) and (O2):

(O1') $\lambda w \lambda t\ [{}^0\mathrm{B}_{wt}\ {}^0\mathrm{O}\ [\lambda w^* \lambda t^*\ {}^0\exists x\ [{}^0\mathrm{S}_{w^*t^*}\ x]]]$ ^{0}B – *de re*, ^{0}S – *de dicto*, ${}^0\exists$ - "*de dicto*"
(O2') $\lambda w \lambda t\ [{}^0\mathrm{B}_{wt}\ {}^0\mathrm{O}\ [\lambda w^* \lambda t^*\ [{}^0\mathrm{E}_{w^*t^*}\ {}^0\mathrm{S}]]]$ ^{0}B – *de re*, ^{0}S – *de dicto*, ^{0}E - *de dicto*

To analyse (O3) and (O4), let us denote the property of being believed by Ortcutt to be a spy by BOS. Auxiliary analysis of (O3) and (O4) is:

(O3') $\lambda w \lambda t\ {}^0\exists x\ [{}^0\mathrm{BOS}_{wt}\ x]$ ^{0}BOS – *de re*, ${}^0\exists$ - "*de re*"
(O4') $\lambda w \lambda t\ [{}^0\mathrm{E}_{wt}\ {}^0\mathrm{BOS}]$ ("BOSs exist") ^{0}E - *de re*, ^{0}BOS – *de dicto*

We can see that the existential constituent ^{0}E occurs in the *de dicto* supposition in (O2') and in the *de re* supposition in (O4'). Using our general characterisation of the *de dicto / de re* distinction (the beginning of Section 4), we can say that the same distinction is displayed by the ${}^0\exists$ component: "*de dicto*" in (O1') – the truth value of the proposition constructed by (O1') in *w, t* – reporter's perspective – does not depend on the value of ${}^0\exists$, Ortcutt can believe that someone is a spy even if there are none, whereas in (O3') the truth value of the constructed proposition does depend on the value of ${}^0\exists$ - "*de re*", $\exists$ is used.

Note: It is easy to see that (O1') and (O2') are equivalent. Substituting the above construction of the existence property for ^{0}E in (O2') and performing an equivalent β-reduction, we obtain (O1'). The same checking shows that (O3') and (O4') are equivalent. (In both cases there is no problem with partiality, because ^{0}S as well as ^{0}BOS, constructing properties, cannot be improper.)

Concluding this analysis, it remains to construct the property BOS -
$\lambda w \lambda t\ \lambda x\ [{}^0\mathrm{Bel}_{wt}\ {}^0\mathrm{O}\ [\lambda w \lambda t\ [{}^0\mathrm{S}_{wt}\ x]]]$ and we get

(O3'') $\lambda w \lambda t\ [{}^0\exists x\ [\lambda w \lambda t\ \lambda x\ [{}^0\mathrm{Bel}_{wt}\ {}^0\mathrm{O}\ [\lambda w \lambda t\ [{}^0\mathrm{S}_{wt}\ x]]]]_{wt}\ x]$ β_i-reduction:
(O3''') $\lambda w \lambda t\ {}^0\exists x\ [\ {}^0\mathrm{Bel}_{wt}\ {}^0\mathrm{O}\ [\lambda w \lambda t\ [{}^0\mathrm{S}_{wt}\ x]]\ ]$
(O4'') $\lambda w \lambda t\ [{}^0\mathrm{E}_{wt}\ [\lambda w \lambda t\ \lambda x\ [{}^0\mathrm{Bel}_{wt}\ {}^0\mathrm{O}\ [\lambda w \lambda t\ [{}^0\mathrm{S}_{wt}\ x]]]]\ ]$.

The *de dicto* and *de re* readings are, of course, not equivalent. Quine's original sentence is not weakly homonymous. Ortcutt can believe that spies exist without believing about somebody that they are a spy.

7. De dicto / de re modalities

When defining *concept* as the meaning of a reasonable expression and the QUID relation as an equivalence of closed constructions pointing to the same concept, we enriched Materna's original definition by the "innocent" β_i-reduction, but warned against a "general" β-reduction. Some reasons have been stated above: Substituting a composed construction for a variable may change the supposition (*de dicto / de re*). Hence β-reduction is in a way too "strong" a transformation switching to another meaning. What is even worse (see Section 5.2), β-reduction is generally *not* an equivalent transformation when working with partial functions. In this section we will analyse another example of a non-equivalent β-reduction, namely the case of combining partial functions and "totalising" quantifiers. Consider the sentence

(FK) The King of France might not have been a king.

A standard analysis using the modal operator ◇ ('possibly') might be, e.g., [Dummett 1981]:
$(\lambda x \diamond \neg \mathrm{K}x)\ (\iota y\ \mathrm{K}y)$ (The class of those who were possibly not a king contains the King)
Now there is a logical problem: Applying the β-rule of λ-calculi we get
$\diamond \neg \mathrm{K}(\iota y\ \mathrm{K}y)$ thus obtaining a contradiction, whereas (FK) is a meaningful sentence (that was true in some period before 1789). Dummett states an *ad hoc* principle that gets no support from standard logic
When a 'modal expression' is applied to a definite description, the β-rule cannot be applied, and he is not able to explain why. In [Materna to appear] this fact is correctly explained by Dummett's negligence of the intensional character of the definite description, thus committing the collision of variables. Materna proposes an analysis by correctly using the β-rule (renaming variables *w* standing for possible worlds). We will show that even such an analysis is not correct, though it does not lead to the impossible (contradictory) proposition.
There are four possible readings of the sentence (FK), two *de dicto* and two *de re*, but only the *de re* readings are plausible (and are probably those intended).
(DD1) It is possible that the King of France is not a king.
(DD2) It is not necessary that the King of France is a king.
(DR1) The King of France is possibly not a king.
(DR2) The King of France is not necessarily a king.
Using for the sake of simplicity the simple concept ^{0}KF to construct the office KF of the King of France, a $\iota_{\tau\omega}$-object, K(ing) / $(o\iota)_{\tau\omega}$, our analysis is:
(DD1') $\lambda w \lambda t\ {}^0\exists w^*\ {}^0\exists t^*\ [{}^0\neg\ [{}^0\mathrm{K}_{w^*t^*}\ {}^0\mathrm{KF}_{w^*t^*}]]$ (^{0}K, ^{0}KF – *de dicto*)
Constructs the impossible proposition, false in all *w,t* pairs (it is impossible that the King of France were not a king *"in the same world, time"*)
(DD2') $\lambda w \lambda t\ [{}^0\neg\ {}^0\forall w^*\ {}^0\forall t^*\ [[{}^0\mathrm{K}_{w^*t^*}\ {}^0\mathrm{KF}_{w^*t^*}]]$ (^{0}K, ^{0}KF – *de dicto*)
Constructs the necessary proposition, true in all *w,t* pairs
(the composition $[{}^0\mathrm{K}_{w^*t^*}\ {}^0\mathrm{KF}_{w^*t^*}]$, the King is a king, is *almost* true so to speak, i.e. true in all those *w*,t** pairs where $^0\mathrm{KF}_{w^*t^*}$ is not *v*-improper, but in the other *w*,t** pairs it is *v*-improper, so that $^0\forall w^*\ {}^0\forall t^*\ [[{}^0\mathrm{K}_{w^*t^*}\ {}^0\mathrm{KF}_{w^*t^*}]]$ is false).
(DR1') $\lambda w \lambda t\ [\lambda x\ [{}^0\exists w^*\ {}^0\exists t^*\ [{}^0\neg\ [{}^0\mathrm{K}_{w^*t^*}\ x]]]\ {}^0\mathrm{KF}_{wt}\ ]$ (^{0}K – *de dicto*, ^{0}KF – *de re*)

It constructs a properly *partial* proposition P which was true in the actual world in some period before the year 1789, but which does not have any truth value now (because $^0\mathrm{KF}_{wt}$ is improper). If P had any truth-value, the King of France would have to exist.

(DR2') $\lambda w \lambda t\, [\lambda x\, [^0\neg\, [^0\forall w^*\, ^0\forall t^*\ [^0K_{w^*t^*}\, x]]]\ ^0KF_{wt}\,]$ (^0K – *de dicto*, ^0KF – *de re*)

It constructs the same proposition P as DR1'.

We can see that (DR1'), as well as (DR2') are adequate analyses of our sentence (FK). They are equivalent, but not quasi-identical, i.e. (FK) is *weakly homonymous.*

But performing a β-reduction, we obtain:

(DR1β) $\lambda w \lambda t\, [^0\exists w^*\, ^0\exists t^*\, [^0\neg\, [^0K_{w^*t^*}\, ^0KF_{wt}\,]]]$ (^0K – *de dicto*, ^0KF – *de re*)

It constructs a *total* proposition P', different from P. The proposition P' "behaves" in the same way as P in those *w,t* pairs where $^0KF_{wt}$ is a proper construction by returning true. But P' is simply *false* in those *w, t* pairs where $^0KF_{wt}$ is *v*-improper (for instance, in the actual possible world now). The class of those *w*,t** for which

$[^0\neg\, [^0K_{w^*t^*}\, ^0KF_{wt}\,]]$ holds is not non-empty, it is again that "degenerated" class, because

$[^0\neg\, [^0K_{w^*t^*}\, ^0KF_{wt}\,]]$ is *v*-improper, and $^0\exists$ returns false.

(DR2β) $\lambda w \lambda t\, [^0\neg\, [^0\forall w^*\, ^0\forall t^*\, [^0K_{w^*t^*}\, ^0KF_{wt}\,]]]$ ^0K – *de dicto*, ^0KF – *de re*

It constructs another *total* proposition P'', the necessary one this time, different from P, P'. The proposition P'' "behaves" in the same way as P in those *w,t* pairs where $^0KF_{wt}$ is a proper construction, it returns true. But P'' is simply also *true* in those *w, t* pairs where $^0KF_{wt}$ is *v*-improper (for instance, in the actual possible world now).

We can see that neither (DR1β) nor (DR2β) is correct as a semantic analysis.

Note: The above example illustrates also an important fact about partial functions, namely that the ***De Morgan laws are not valid when using partial functions.***

For instance the claim that there exists a pair of natural numbers such that their ratio is not a rational number is *false*, whereas the claim that it is not true that for all the pairs of natural numbers it holds that their ratio is a rational number is *true.* Formally,

(Rat / (οτ) - the class of rational numbers, Nat / (οτ) - the class of natural numbers, variables *m, n* ranging over τ)

(E) $[\ ^0\exists\ \lambda\, m\, n\, ([^0Nat\ m] \wedge [^0Nat\ n] \wedge [\neg\, [^0Rat\, [^0\!:\ m\ n]]])\ ]$ is not equivalent to

(G) $[\ \neg\, [^0\forall\ \lambda\, m\, n\, (\, ([^0Nat\ m] \wedge [^0Nat\ n]) \supset [^0Rat\, [^0\!:\ m\ n]]\ )]\ ]$.

We will show that *(E) constructs false, whereas (G) constructs true.*

The composition $([^0Nat\ m] \wedge [^0Nat\ n] \wedge [\neg\, [^0Rat\, [^0\!:\ m\ n]]])$ *v*-constructs false for all the valuations *v* that do not assign 0 to *n,* and it is *v*-improper for *v* assigning 0 to *n.* Hence the class constructed by $\lambda\, m\, n\, ([^0Nat\ m] \wedge [^0Nat\ n] \wedge [\neg\, [^0Rat\, [^0\!:\ m\ n]]])$ is not non-empty and $^0\exists$ returns false, while (E) constructs false. On the other hand, the composition $(\, ([^0Nat\ m] \wedge [^0Nat\ n]) \supset [^0Rat\, [^0\!:\ m\ n]]\ )$ *v*-constructs true for all the valuations *v* that do not assign 0 to *n,* and it is *v*-improper for *v* assigning 0 to *n.* Hence the class constructed by $\lambda\, m\, n\, (\, ([^0Nat\ m] \wedge [^0Nat\ n]) \supset [^0Rat\, [^0\!:\ m\ n]]\ )$ is not the whole type τ; therefore, $^0\forall$ returns false and the entire (G) constructs true.

The above findings can be condensed into the following:

β-reduction is generally not an equivalent transformation when working with partial functions and must not be carelessly applied.

8. Conclusion

In this paper we have examined and solved some traditional hard nuts typical of the enterprise of the logical analysis of natural language, namely the problem of an exact definition of synonymy, homonymy, equivalence, the *de dicto / de re* distinction, and last but not least, *de dicto / de re* attitudes, existential claims and *de dicto / de re* modalities. Solving these problems we used the apparatus of Transparent Intensional logic (TIL), which enabled us to explicitly distinguish the ”two worlds”, reporter’s perspective and believer’s perspective, by means of explicit intensionalisation, avoiding thus the need for any non-standard operators like the independence indicator or backward-looking operators. As a side-effect of our investigations, we presented an adjustment of a theory of concept as the meaning of a reasonable natural language expression. Hence this contribution can be characterised as a new approach to the above problems using in a way traditional means.

We have also demonstrated (and solved) many problems connected with our functional approach, first of all concerning *partial functions*. It is a brute fact that there are partial functions in mathematics, for instance, the function of dividing (unless we unnaturally restrict the domain of the function), and that we have to be able to deal with them. Intensions, being partial functions, form a part of the modelling of reality. But being still a model, does it mean that we can do with it whatever we wish? Does anything require, in and by itself, us to adopt partial functions in order to reflect ‘holes’ in reality? Convenience might also dictate that we adopted totally defined intensions *tout court,* which would, after all, be easier. Strawson’s remarks on ontological presupposition are substantial only if one has already decided that one’s intensions are supposed to reflect reality quite faithfully. There might have been a King of France in the actual world now, but there is none. If we modelled this situation by a total function, we would simply have to supply an individual as a value of the function. Who should it be? How would such an analysis comply with the demand that it must not allow us to deduce any non-adequate consequences of our statements? Intensions are just explications, not the real things, but they should be as faithful as possible.

We are aware of the fact that much has been done in this area, and that a general survey of the ”state-of-the-art” is here missing, which might be considered to be a shortcoming of this paper. However, this is always a problem when presenting something new and such a summary of and comparison with the traditional approaches has been out of the scope of this paper.

Anyway, since it should be done, a future study is planned with Bjorn Jespersen that will compare the philosophical assumptions of this paper with those of traditional approaches.

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References

[Barwise, Perry 1983] J. Barwise, J. Perry: *Situations and Attitudes.* MIT Press, Cambridge, Mass.

[Bolzano 1837] B. Bolzano: *Wissenschaftslehre I, II*. Sulzbach.

[Carnap 1947] R. Carnap: *Meaning and Necessity.* Chicago UP.

[Castaneda 65] H.N. Castaneda: On the Logic of Attributions of Self-Knowledge to Others. *The Journal of Philosophy*, 65, pp. 439-456.

[Chisholm 1976] R. Chisholm: Knowledge and Belief: 'De dicto' and 'de re'. *Philosophical Studies* 29 (1976), 1-20.
[Church 1954] A. Church: Intensional Isomorphism and Identity of Belief. *Philosophical Studies*, Vol. 5, pp. 65-73.
[Cresswell 1975] M.J. Cresswell: 'Hyperintensional Logic'. *Studia Logica* 34, pp.25-38.
[Cresswell 1985] M.J. Cresswell: *Structured meanings.* MIT Press, Cambridge, Mass.
[Dummett 1981] M. Dummett: *The Interpretation of Frege's Philosophy.* Duckworth, London.
[Duží 1999] M. Duží: Existential Quantification into Intensional Contexts. The Logica Yearbook 1999, Filosofia, Prague 2000, pp. 258-272.
[Duží 1999a] M. Duží: Knowing that ... *11th International Congress of Logic, methodology and Philosophy of Science*, Cracow, Poland, p. 101.
[Duží 2000] M. Duží: De re vs. De dicto. *Organon F*, SAV Bratislava, 2000, VII, 4, pp. 365-378.
[Duží, Materna 1994] M. Duží, P. Materna: Non-reasonable sentences. *Logica'94,* Filosofia, Prague 1994, pp. 107-124.
[Duží, Materna 2000] M. Duží, P. Materna: Propositional attitudes revised. *The Logica Yearbook 2000*, Filosofia, Prague, 2001, pp.163-174.
[Duží 2003a] M. Duží: *Do we have to deal with partiality?* In Miscellania Logica, Prague 2003, Tom V, 45-76.
[Duží 2003b] M. Duží: Notional attitudes (On wishing, seeking and finding). In *Organon F*, Philosophical Journal, Bratislava 2003, vol. X, No 3, 237-260
[Fitch 1981] G.W. Fitch: Names and the 'de re - de dicto' distinction. *Philosophical Studies* 39 (1981), 25-34.
[Frege 1884] G. Frege: *Die Grundlagen der Arithmetik.* W. Koebner, Breslau.
[Frege 1892] G. Frege: "Über Sinn and Bedeutung". *Zeitschrift f. Philosophie und philosophische Kritik 100,* pp.25-50.
[Hintikka, Sandu 1989] J. Hintikka, G. Sandu: Informational Independence as a Semantical Phenomenon. In J.E. Fenstad *et el* (eds.), *Logic, Methodology and Philosophy of Science*, Elsevier, Amsterodam 1989, pp. 571-589.
[Hintikka, Sandu 1996] J. Hintikka, G. Sandu: A revolution in Logic? *Nordic Journal of Philosophical Logic*, Vol.1, No.2, pp. 169-183.
[Horák 2001] A. Horák: *The Normal Translational Algorithm in Transparent Intensional Logic for Czech*. PhD thesis, Masaryk University of Brno, 2001.
[Jespersen 1999] B. Jespersen: On Seeking and Finding. *The Logica Yearbook 1998*, Filosofia, Prague, 1999, pp. 61-72.
[Jespersen 2000] B. Jespersen:*Attitudes and Singular Reference in Transparent Intensional Logic.* PhD thesis, Masaryk University of Brno.
[Jespersen 2001] B. Jespersen: Is Paradoxical Belief Possible? Yes and No! *The Logica Yearbook 2001*, Filosofia, Prague, to appear.
[Kaplan 1986] Opacity. In: W.V. Quine (L.Hahn, ed.) La Salle: Open Const, 1986, 229-89.
[Kaplan 1968] Quantifying in. *Synthese* 19 (1968-69), pp. 178-214.
[Larson, Ludlow 1995] R. Larson, P. Ludlow: 'Interpreted Logical Forms'. *Synthese* 95, pp. 305-355.
[Materna 1998] P. Materna: *Concepts and Objects.* Acta Philosophica Fenica, vol. 63, 1998.
[Materna 1999] P. Materna: Simple concepts and simple expressions. *Logica'99*,

FILOSOFIA, Prague 2000, 245-257.

[Materna 1999] Materna, Pavel: Actuality and Possibility from the Viewpoint of Transparent Intensional Logic. In: A.Rojszczak, A.Cachro, G.Kurczewski, eds.: *Philosophical Dimensions of Logic and Science.* Kluwer Academic Publishers, Dordrecht / Boston / London, 289-296, 2003

[McKinsey 1999] M. McKinsey: The Semantics of Belief Ascriptions. *NOUS* 33:4 (1999), 519-557.

[Montague 1974] R. Montague: *Formal Philosophy: Selected papers of R. Montague* (ed. R. Thomason), Yale University Press, New Haven.

[Quine 1956] W.V.O. Quine: Quantifiers and propositional Attitudes. *Journal of Philosophy* 53, 177-187.

[Quine 1960] W.V.O. Quine: *Word and Object.* MIT Press, Cambridge, Mass.

[Quine 1990] W.V.O. Quine: *The Pursuit of Truth*. Harvard UP, Cambridge, Mass.

[Quine 1995] W.V.O. Quine: *From Stimulus to Science*, Harvard University Press,Cambridge (Mass.).

[Russell 1905] B. Russell: On Denoting, *Mind*, 14, 479-493.

[Stalnaker 1999] R.C. Stalnaker: The problem of logical omniscience I. In: *Context and Content* (pp.241-254). Oxford UP 1999.

[Strawson 1950] P.F. Strawson: On Referring. *Mind* 59, pp.320-344.

[Tichý 1971] P. Tichý: An Approach to Intensional Analysis. *NOUS*, Volume 5, Issue 3, pp. 273-297.

[Tichý 1978] P. Tichý: De dicto vs. de re. *Philosophia* 8, 1978, pp. 1-16.

[Tichý 1979] P. Tichý: Existence and God. *Journal of Philosophy* 76, 1979, pp. 403-420.

[Tichý 1986] P. Tichý: The Indiscernibility of Individuals. *Studia Logica* XLV, 3/86, pp. 251-273.

[Tichý 1988] P. Tichý: *The Foundations of Frege's Logic.* De Gruyter.

[Zalta 1988] E.N. Zalta: *Intensional Logic and the Metaphysics of Intensionality.* MIT Press, Cambridge, Mass., London.5